Documentation of the 2005 Version of the MAQS Dynamic Stochastic General Equilibrium Model of the U.S. Economy

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Abstract

This paper presents detailed documentation of the large-scale estimated monetary dynamic general equilibrium model of the US economy that is currently being developed in the Division of Research and Statistics at the Federal Reserve Board.

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This paper provides a detailed documentation dynamic stochastic general equilibrium model currently being developed in the Division of Research and Statistics at the Federal Reserve Board. Section 1 outlines the models production, capital evolution, and preference technologies, section 2 describes the economy's decentralization, and section 3 described equilibrium in the model. (Appendix A presents the equations of the symmetric model and appendix B reports the equations of the symmetric and stationary model.) Section 4 describes the data used in estimating the model and section 5 reports the model's estimation results. (A key for all of the model's variable names and parameters is given in appendixes C to E.)

1 Production, Capital Evolution, and Preferences

In this section we present the production, capital evolution, and preference technologies for our model. The long-run evolution of the economy is determined by differential rates of stochastic growth in the production sectors of the economy, while its short-run dynamics are influenced by various forms of adjustment costs. Adjustment costs to real aggregate variables are captured by the economy's preference, production, and capital evolution technologies presented in this section. Adjustment costs to real sectoral variables and nominal variables are captured in the decentralization of the model presented in the following section.

1.1 The Production Technology

As noted in the previous section our model economy produces four final goods and services:

- Government services, X_t^{cg};
- Housing services, X_t^{ch};
- Slow-growing "consumption" goods and services X_t^{cbi}; and,
- Fast-growing "capital" goods X_t^{kb}.

We consider first the production of the economy's **slow-growing "consumption" goods and services** and **fast-growing "capital" goods**. These final goods are produced by aggregating—according to a Dixit-Stiglitz technology—an infinite number of differentiated inputs. Specifically, final goods production is represented by the function

$$X_t^s = \left(\int_0^1 X_t^s(j)^{\frac{\Theta_t^{x,s}-1}{\Theta_t^{x,s}}} dj \right)^{\frac{\Theta_t^{x,s}}{\Theta_t^{x,s}-1}}, \qquad s = cbi, kb, \tag{1}$$

where the variable $X_t^s(j)$ denotes the quantity of the jth input (obtained from the intermediate goods sector) used to produce final output $s = cbi$ or $s = kb$ while $\Theta_t^{x,s}$ is the stochastic elasticity of substitution between the differentiated intermediate goods inputs used in the production of the consumption or capital goods sectors. Letting $\theta_t^{x,s} \equiv \ln \Theta_t^{x,s} - \ln \Theta_*^{x,s}$ denote the log-deviation of $\Theta_t^{x,s}$ from its steady-state value of $\Theta_*^{x,s}$, we assume that

$$\theta_t^{x,s} = \rho^{\theta,x,s}\theta_{t-1}^{x,s} + \epsilon_t^{\theta,x,s} \tag{2}$$

where $\epsilon_t^{\theta,x,s}$ is an i.i.d. shock process, and $\rho^{\theta,x,s}$ represents the persistence of $\Theta_t^{x,s}$ away from steady-state following a shock to equation (2).

The jth differentiated intermediate good in sector s (which is used as an input in equation 1) is produced by combining each variety of the economy's differentiated labor inputs $\{L_t^s(i,j)\}_{i=0}^1$ with the sector's specific *utilized* non-residential capital stock $K_t^{u,nr,s}(j)$. (Utilized non-residential capital, $K_t^{u,nr,s}(j)$, is equal to the product of *physical* non-residential capital, $K_t^{nr,s}(j)$, and the utilization rate, $U_t^{nr,s}(j)$). A Dixit-Stiglitz aggregator characterizes the way in which differentiated labor inputs are combined to yield a composite bundle of labor, denoted $L_t^s(j)$. A Cobb-Douglas production function then characterizes how this composite bundle of labor is used with capital to produce—given the current level of multifactor productivity MFP_t^s in the sector s—the intermediate good $X_t^s(j)$. The production of intermediate good j is represented by the function:

$$X_t^{m,s}(j) = (K_t^{u,nr,s}(j))^{\alpha}\left(\underbrace{A_t^m Z_t^m A_t^s Z_t^s}_{MFP_t^s} L_t^{x,s}(j)\right)^{1-\alpha} \text{ where } L_t^{x,s}(j) = \left(\int_0^1 L_t^{x,s}(i,j)^{\frac{\Theta_t^{l,s}-1}{\Theta_t^{l,s}}} di\right)^{\frac{\Theta_t^{l,s}}{\Theta_t^{l,s}-1}} s = cbi, kb. \tag{3}$$

The parameter α in equation (3) is the elasticity of output with respect to capital while $\Theta_t^{l,s}$ denotes the stochastic elasticity of substitution between the differentiated labor inputs. Letting $\theta_t^{l,s} \equiv \ln \Theta_t^{l,s} - \ln \Theta_*^{l,s}$ denote the log-deviation of $\Theta_t^{l,s}$ from its steady-state value of Θ_*^{l}, we assume that

$$\theta_t^{l,s} = \rho^{\theta,l,s}\theta_{t-1}^{l,s} + \epsilon_t^{\theta,l,s} \tag{4}$$

where $\epsilon_t^{\theta,l,s}$ is an i.i.d. shock process, and $\rho^{\theta,l,s}$ represents the persistence of $\Theta_t^{l,s}$ away from steady-state following a shock to equation (4).

The level of technology in sector s has four components. The A_t^m and Z_t^m components represent economy-wide technology shocks, while the A_t^s and Z_t^s terms (for $s = cbi, kb$)

represent technology shocks that are specific to either the consumption or capital goods sectors. The A_t technology terms represent shocks that exhibit only transitory movements away from their steady-state unit mean, while the Z_t technology terms represent shocks that exhibit permanent movements in their levels. Specifically, letting $a_t^s \equiv \ln A_t^s$ denote the log-deviation of A_t^s from its steady-state value of unity, we assume that

$$a_t^s = \rho^{a,s} a_{t-1}^s + \epsilon_t^{a,s}, \;\; s = cbi, kb, m \tag{5}$$

where $\epsilon_t^{a,s}$ is an i.i.d. shock process, and $\rho^{a,s}$ represents the persistence of A_t^s away from steady-state following a shock to equation (5). The stochastic process Z_t^s evolves according to

$$\ln Z_t^s - \ln Z_{t-1}^s = \ln \Gamma_t^{z,s} = \ln \left(\Gamma_*^{z,s} \cdot \exp[\gamma_t^{z,s}]\right) = \ln \Gamma_*^{z,s} + \gamma_t^{z,s}, \;\; s = cbi, kb, m \tag{6}$$

where $\Gamma_*^{z,s}$ and $\gamma_t^{z,s}$ are the steady-state and stochastic components of $\Gamma_t^{z,s}$. The stochastic component $\gamma_t^{z,s}$ is assumed to evolve according to

$$\gamma_t^{z,s} = \rho^{z,s} \gamma_{t-1}^{z,s} + \epsilon_t^{z,s}. \tag{7}$$

where $\epsilon_t^{z,s}$ is an i.i.d shock process, and $\rho^{z,s}$ represents the persistence of $\gamma_t^{z,s}$ to a shock. In line with historical experience, we assume a more rapid rate of technological progress in capital goods production by calibrating the steady-state growth rate of the non-stationary component of technology in the capital goods sector above that in the consumption goods sector. That is, $\Gamma_*^{z,k} > \Gamma_*^{z,c} (= 1)$, where an asterisk on a variable denotes its steady-state value.

We assume, as do the BEA, that **housing services**, X_t^{ch}, are produced from the economy's stock of residential capital, K_t^r. The production function for consumer housing services is given by

$$X_t^{ch} = \frac{\Gamma_*^{x,cbi} - \beta(1-\delta^r)}{\beta} \cdot K_t^r,$$

Finally, the growth rate of **government services** output, $H_t^{cg} = X_t^{cg}/X_{t-1}^{cg}$, follows an exogenous auto-regressive process. Specifically, letting $h_t^{cg} = \ln H_t^{cg} - \ln H_*^{cg}$ is assumed to evolve according to

$$h_t^{cg} = \rho^{h,cg} h_{t-1}^{cg} + \epsilon_t^{h,cg}, \tag{8}$$

where $\epsilon_t^{h,cg}$ is an i.i.d shock process, and $\rho^{h,cg}$ represents the persistence of h_t^{cg} to a shock.

1.2 Capital Stock Evolution

As already noted, there are three types of *physical* capital stocks in our model economy:

- Non-residential capital, K_t^{nr};
- Residential capital, K_t^r; and,
- Consumer durables capital K_t^{cd}.

Purchases of the economy's fast-growing "capital" good can be transformed into either non-residential capital, K_{t+1}^{nr}, (that can then be used in the production of either the slow-growing "consumption" good or the fast-growing "capital" good) or into the economy's consumer-durable capital stock, K_{t+1}^{cd}, (from which households derive utility). Purchases of the economy's slow-growing "consumption" good can be transformed into residential capital (from which households derive utility and for which the BEA in constructing the NIPA assume is used for the production of housing services.)

The evolution of the economy's three capital stocks are given below. We assume that there is some stochastic element—reflected in the term A_t^s, for $s = nr$, cd, and r—in the capital accumulation process. These shocks exhibit only transitory movements from their steady-state unit mean. Letting $a_t^s \equiv \ln A_t^s$ denote the log-deviation of A_t^s from its steady-state value of unity, we assume that:

$$a_t^s = \rho^{a,s} a_{t-1}^s + \epsilon_t^{a,s}, \;\; s = nr, cd, r. \tag{9}$$

We also asssume that not all investment expenditure results in productive capital, since some fraction is absorbed by adjustment costs in the process of installation:

$$\begin{aligned} K_{t+1}^{nr}(k) = & (1-\delta^{nr})K_t^{nr}(k) + A_t^{nr} E_t^{nr}(k) \\ & - \frac{100 \cdot \chi^{nr}}{2} \left(\frac{E_t^{nr}(k) - \eta^{nr} E_{t-1}^{nr}(k) \Gamma_t^{x,kb} - (1-\eta^{nr}) \widetilde{E}_*^{nr} Z_t^m Z_t^{kb}}{K_t^{nr}} \right)^2 K_t^{nr} \end{aligned} \tag{10}$$

$$\begin{aligned} K_{t+1}^{cd}(k) = & (1-\delta^{cd})K_t^{cd}(k) + A_t^{cd} E_t^{cd}(k) \\ & - \frac{100 \cdot \chi^{cd}}{2} \left(\frac{E_t^{cd}(k) - \eta^{cd} E_{t-1}^{cd}(k) \Gamma_t^{x,kb} - (1-\eta^{cd}) \widetilde{E}_*^{cd} Z_t^m Z_t^{kb}}{K_t^{cd}} \right)^2 K_t^{cd} \end{aligned} \tag{11}$$

$$\begin{aligned} K_{t+1}^{r}(k) = & (1-\delta^{r})K_t^{r}(k) + A_t^{r} E_t^{r}(k) \\ & - \frac{100 \cdot \chi^{r}}{2} \left(\frac{E_t^{r}(k) - \eta^{r} E_{t-1}^{r}(k) \Gamma_t^{x,cbi} - (1-\eta^{r}) \widetilde{E}_*^{r} Z_t^m (Z_t^{kb})^{\alpha} (Z_t^{cbi})^{1-\alpha}}{K_t^{r}} \right)^2 K_t^{r}. \end{aligned} \tag{12}$$

The parameter δ^s denotes the depreciation rate for either the non-residential ($s = nr$), consumer durables ($s = cd$), or residential ($s = r$) capital stocks. The term $\widetilde{E}_*^s$ denotes the

value of steady-state non-residential ($s = nr$), consumer durables ($s = cd$), or residential ($s = r$) investment spending normalized by the permanent component of technology so as to be constant in the steady state. Note that investment adjustment costs are zero for non-residential capital when $\frac{E_t^{nr}}{Z_t^m Z_t^{kb}} = \frac{E_{t-1}^{nr}}{Z_{t-1}^m Z_{t-1}^{kb}} = \widetilde{E}_*^{nr}$ but rise to above zero, at an increasing rate, as non-residential investment growth moves further away from this. The costs for altering non-residential investment depend on both the level of (growth-adjusted) investment spending from the preceding period as well as the steady-state level of investment spending. The parameter χ^{nr} governs how quickly these costs increase away from the steady-state. The relative values of $\frac{E_t^{cd}}{Z_t^m Z_t^{kb}}$, $\frac{E_{t-1}^{cd}}{Z_{t-1}^m Z_{t-1}^{kb}}$, and $\widetilde{E}_*^{cd}$ have similar implications for the adjustment costs entailed in the accumulation of consumer durables capital, as do the relative values of $\frac{E_t^r}{Z_t^m (Z_t^{kb})^\alpha (Z_t^{cbi})^{1-\alpha}}$, $\frac{E_{t-1}^r}{Z_{t-1}^m (Z_{t-1}^{kb})^\alpha (Z_{t-1}^{cbi})^{1-\alpha}}$, and $\widetilde{E}_*^r$ for the accumulation of residential capital. Similarly, the values of the parameter χ^{cd} and χ^r govern how quickly these costs increase away from the steady-state.

1.3 Preferences

The ith household derives utility from four sources:

- Its purchases of the consumer non-durable goods and non-housing services, $E_t^{cnn}(i)$;
- The flow of services from its stock of consumer-durable capital, $K_t^{cd}(i)$;
- The flow of services from its stock of residential capital $K_t^r(i)$; and
- Its leisure time, which is equal to what remains of its time endowment after $L_t^{cbi}(i) + L_t^{kb}(i)$ hours are spent through working.

The preferences of household i are separable over all of the arguments of its utility function. The utility that the household derives from the three components of its goods and services consumption is influenced by its habit stock for each of these consumption components. Household i's habit stock for its consumption of non-durable goods and non-housing services, is equal to a factor h^{cnn} multiplied by its consumption last period $E_{t-1}^{cnn}(i)$). The household's habit stock for its other components of consumption is defined similarly. In summary, the preferences of household i are represented by the utility function:

$$\mathcal{E}_0 \sum_{t=0}^{\infty} \beta^t \Big\{ \varsigma^{cnn} \Xi_t^{cnn} \ln(E_t^{cnn}(i) - h^{cnn} E_{t-1}^{cnn}(i)) + \varsigma^{cd} \Xi_t^{cd} \ln(K_t^{cd}(i) - h^{cd} K_{t-1}^{cd}(i)) + \varsigma^r \Xi_t^r \ln(K_t^r(i) - h^r K_{t-1}^r(i)) - \varsigma^l \Xi_t^l \frac{(L_t^{cbi}(i) + L_t^{kb}(i))^{1+\nu}}{1+\nu} \Big\}. \quad (13)$$

The parameter β is the household's discount factor, ν denotes its inverse labor supply elasticity, while ς^{cnn}, ς^{cd}, ς^{r}, and ς^{l} are scale parameter that tie down the ratios between the household's consumption components. The stationary, unit-mean, stochastic variables Ξ_t^{cnn}, Ξ_t^{cd}, Ξ_t^{r}, and Ξ_t^{l} represent aggregate shocks to the household's utility of its consumption components and its disutility of labor. Letting $\xi_t^x \equiv \ln \Xi_t^x - \ln \Xi_*^x$ denote the log-deviation of Ξ_t^x from its steady-state value of Ξ_*^x, we assume that

$$\xi_t^x = \rho^{\xi,x} \xi_{t-1}^x + \epsilon_t^{\xi,x}, \qquad x = cnn, cd, r, l. \tag{14}$$

The variable $\epsilon_t^{\xi,x}$ is an i.i.d. shock process, and $\rho^{\xi,x}$ represents the persistence of Ξ_t^x away from steady-state following a shock to equation (14).

2 The Decentralized Economy

We assume the following decentralization of the economy:

- The representative, perfectly competitive firm that exists in each of the economy's two final-goods producing sectors purchases intermediate inputs from the continuum of intermediate goods producers to produce the sector's final goods output.
- Firms in the continuum of intermediate goods producers in both of the economy's intermediate-goods producing sectors rent non-residential capital at a set utilization rate from the representative capital owner, and differentiated types of labor from households. Because each intermediate goods producer is a monopolistically competitive supplier of its own output, it is able to set the price at which it sells its product.
- The representative, perfectly competitive capital owner purchases the the output from the fast-growing "capital" final-goods sector and transforms it into either non-residential capital or consumer durables capital. The representative capital owner also purchases the output from the slow-growing "consumption" final-goods and transforms it into residential capital.
- Households in the continuum of consumers purchase the output of the slow-growing "consumption" goods sector, and rent consumer durables capital and residential capital from the capital owner. Because each households is a monopolistically competitive supplier of its own labor, it is able to set the wage at which it supplies its labor services.

- The monetary authority sets the nominal interest rate given an interest rate feedback rule with smoothing.
- The fiscal authority raises taxes in a lump-sum fashion to cover the subsidy outlays that is makes to monopolistically competitive intermedite-goods producing firms and households (in their capacity of labor suppliers) to induce them to supply (in the absence of sticky-prices) at the pareto optimal level. Taxes are also raised to cover the government's purchases of goods and services from the business and institutions sector.
- Foreigners, who are not modeled in any detail, pay lump-sum transfers to households equal to the negative of net exports.

2.1 Consumption and Capital Final Goods Producers

The representative, perfectly competitive firm in the consumption good sector owns the production technology described in equation (1) for $s = cbi$, while the representative, perfectly competitive firm in the capital goods sector owns the same technology for $s = kb$.

The final-good producing firm in sector s takes as given the prices $\{P_t^s(j)\}_{j=0}^1$ set by each intermediate good producing firm for its differentiated output, and choose $\{X_t^s(j)\}_{j=0}^1$ to minimize its production costs subject to the aggregator function. Specifically, the final-good producer in sector s solves the cost-minimization problem of:

$$\min_{\{X_t^s(j)\}_{j=0}^1} \int_0^1 P_t^s(j) X_t^s(j) dj \text{ subject to } \left(\int_0^1 (X_t^s(j))^{\frac{\Theta_t^{x,s}-1}{\Theta_t^{x,s}}} dj \right)^{\frac{\Theta_t^{x,s}}{\Theta_t^{x,s}-1}} \geq X_t^s, \text{ for } s = cbi, kb. \tag{15}$$

The cost-minimization problems solved by firms in the economy's consumption and capital goods producing sectors imply demand functions for each intermediate good that are given by $X_t^s(j) = (P_t^s(j)/P_t^s)^{-\Theta_t^{x,s}} X_t^s$. The variable P_t^s, which denotes the aggregate price level in the final goods sector s, is defined by $P_t^s = (\int_0^1 (P_t^s(j))^{1-\Theta_t^{x,s}} dj)^{\frac{1}{1-\Theta_t^{x,s}}}$.

2.2 Consumption and Capital Intermediate Goods Producers

Each intermediate-good producing firm $j \in [0, 1]$ and $s = c, k$ owns the production technology described in equation (3). In describing the intermediate good producing firm's problem it is convenient to split it into three separate stages.

In the first stage of the problem firm j in sector s, taking as given the wages $\{W_t^s(i)\}_{i=0}^1$ set by each household for its variety of labor supplied to sector s, chooses $\{L_t^s(i,j)\}_{i=0}^1$ to minimize the cost of attaining the aggregate labor bundle $L_t^s(j)$ that it will ultimately need for production. Specifically, the intermediate firm j solves:

$$\min_{\{L_t^s(i,j)\}_{i=0}^1} \int_0^1 W_t^s(i) L_t^s(i,j) di \text{ subject to } \left(\int_0^1 \left(L_t^s(i,j)\right)^{\frac{\Theta_t^{l,s}-1}{\Theta_t^{l,s}}} di \right)^{\frac{\Theta_t^{l,s}}{\Theta_t^{l,s}-1}} \geq L_t^s(j), \text{ for } s = c, k. \tag{16}$$

This cost-minimization problem undertaken by each intermediate good producing firm implies that the demand in sector s for type i labor is $L_t^s(i) = \int_0^1 L_t^s(i,j)dj = (W_t^s(i)/W_t^s)^{-\Theta_t^{l,s}} \times \int_0^1 L_t^s(j)dj$ where W_t^s denotes the aggregate wage for labor supplied to sector s, defined by $W_t^s = (\int_0^1 (W_t^s(x))^{1-\Theta_t^{l,s}} dx)^{\frac{1}{1-\Theta_t^{l,s}}}$.

In the second stage of the problem firm j in sector s, taking as given the aggregate sector s wage W_t^s and the sector s rental rate on capital $R_t^{nr,s}$, chooses aggregate labor $L_t^s(j)$ and capital $K_t^{u,nr,s}(j)$ to minimize the costs of attaining its desired level of output $X_t^s(j)$. Specifically, firm j in sector s solves:

$$\begin{aligned} &\min_{\left\{L_t^s(j), K_t^{u,nr,s}(j)\right\}} W_t^s L_t^s(j) + R_t^{nr,s} K_t^{u,nr,s}(j) \\ &\text{subject to } \left(A_t^m Z_t^m A_t^s Z_t^s L_t^s(j)\right)^{1-\alpha} \left(K_t^{u,nr,s}(j)\right)^{\alpha} \geq X_t^s(j), \text{ for } s = cbi, kb. \end{aligned} \tag{17}$$

Since each intermediate goods firm produces its own differentiated variety of output $X_t^s(j)$, it is able to set its price $P_t^s(j)$. It does this taking into account the demand schedule for its output that it faces from the final-goods sector s, $X_t^s(j) = (P_t^s(j)/P_t^s)^{-\Theta_t^{x,s}} X_t^s$, as well as the adjustment costs that it faces in altering its price. These adjustment costs are introduced to the model through the assumption that the act of altering prices absorbs some of the firm's profits. Specifically, we assume that the firms profits in period t are given by:

$$\begin{aligned} \mathit{Profits}_t^s(j) = &\left(1+\sigma^{p,s}\right) P_t^s(j) X_t^s(j) - MC_t^s(j) X_t^s(j) - \mathit{Taxes}_t^s(j) \\ &- \frac{100 \cdot \chi^{p,s}}{2} \left(\frac{P_t^s(j)}{P_{t-1}^s(j)} - \eta^{p,s} \Pi_{t-1}^{p,s} - (1-\eta^{p,s}) \Pi_*^{p,s} \right)^2 P_t^s X_t^s, \text{ for } s = cbi, kb, \end{aligned} \tag{18}$$

where $\sigma^{p,s} = (\Theta_*^{p,s} - 1)^{-1}$ is a subsidy to production that is set to ensure that the economy's level of steady-state output is Pareto optimal, $MC_t^s(j)$ represents the marginal cost of producing a unit of $X_t^s(j)$ and $\mathit{Taxes}_t^s(j) = \sigma^{p,s} P_t^s X_t^s$, which implies that the government raises (in a lump-sum fashion) the revenue needed to payout the production subsidy. The

parameter $\eta^{p,s}$ reflects the importance of lagged inflation relative to steady-state inflation in determining adjustment costs and $\Pi_*^{p,s}$ denotes the steady-state rate of sector s price inflation. The parameter $\chi^{p,s}$ scales linearly the magnitude of the adjustment cost term; in a flexible price model $\chi^{p,s}$ wouble be equal to zero. In what follows we consider the most general case of the firm's profit-maximization problem, that is, the one in which there are positive price adjustment costs; the first-order conditions implied from a model with sticky prices can be trivially converted to those of the model with flexible prices by simply setting the price adjustment cost parameter equal to zero.

In the profit-maximizing part of its problem, the intermediate-good producing firm j, taking as given the marginal cost $MC_t^s(j)$ for producing $X_t^s(j)$, the aggregate sector s price level P_t^s, and final output X_t^s by sector s, chooses its price $P_t^s(j)$ to maximize the present discounted value of its profits, described in equation (18), subject to the demand curve it faces for its differentiated output. Since the intermediate firms are ultimately owned by the economy's households, intermediate producers act on their behalf when making their profit-maximizing price-setting decisions. For this reason the intermediate firms value their revenues and costs across time exactly as the household would value them. Consequently, the discount factor that is relevant when comparing nominal revenues and costs in period t with those in period $t+j$ is $\beta^j \frac{\Lambda_{t+j}^{cnn}/P_{t+j}^{cbi}}{\Lambda_t^{cnn}/P_t^{cbi}}$, where $\Lambda_t^{cnn} = \int_0^1 \Lambda_t^{cnn}(i)di$. The variable $\Lambda_t^{cnn}(i)$ denotes the household i's marginal utility of consumption in period t, which implies that Λ_t^{cnn} is the average marginal utility of consumption across households. Put formally, the sector s intermediate-good producing firm's profit-maximization problem is:

$$\max_{\{P_t^s(j),X_t^s(j),X_t^s(j)\}_{t=0}^{\infty}} \mathcal{E}_0 \sum_{t=0}^{\infty} \beta^t \frac{\Lambda_t^{cnn}}{P_t^{cbi}} \left\{ (1+\sigma^{p,s}) P_t^s(j) X_t^s(j) - MC_t^s(j) X_t^s(j) - \sigma^{p,s} P_t^s X_t^s \right.$$
$$\left. - \frac{100 \cdot \chi^{p,s}}{2} \left(\frac{P_t^s(j)}{P_{t-1}^s(j)} - \eta^{p,s} \Pi_{t-1}^{p,s} - (1-\eta^{p,s}) \Pi_*^{p,s} \right)^2 P_t^s X_t^s \right\}$$
$$\text{subject to } X_\tau^s(j) = \left(\frac{P_\tau^s(j)}{P_\tau^s} \right)^{-\Theta_\tau^{x,s}} X_\tau^s$$
$$\text{for } \tau = 0, 1, ..., \infty, \text{ and } s = cbi, kb. \tag{19}$$

2.3 Capital Owners

Capital owners possess the technologies described in equations (10) and (11) for transforming the economy's fast-growing "capital" good into either non-residential capital, K_{t+1}^{nr}, (that is then rented out to intermediate firms for use in the production process) or into the

economy's consumer-durable capital stock, K_{t+1}^{nr}, (that is then rented out to households). The also possess the technology described in equation (12) for transforming the economy's slow-growing "consumption" good into the economy's residential capital stock K_{t+1}^{nr}, (that is also rented out to households).

We assume that capital owners face a cost in moving non-residential capital between the two intermediate goods producing sectors of the economy (but do not encounter any cost in moving capital between firms within the same sector). Specifically, the relationship between the aggregate capital stock $K_t^{nr}(k)$ and the capital stocks used in the consumption and capital intermediate goods producting sectors, that is $K_t^{nr,cbi}(k)$ and $K_t^{nr,kb}(k)$, is given by:

$$\begin{aligned} &K_t^{nr,cbi}(k)+K_t^{nr,kb}(k) \\ &=K_t^{nr}(k)-\frac{100\cdot\chi^k}{2}\left(\frac{K_t^{nr,cbi}(k)}{K_t^{nr,kb}(k)}-\eta^k\frac{K_{t-1}^{nr,cbi}}{K_{t-1}^{nr,kb}}-(1-\eta^k)\frac{\widetilde{K}_*^{nr,cbi}}{\widetilde{K}_*^{nr,kb}}\right)^2\frac{K_t^{nr,kb}}{K_t^{nr,cbi}}\cdot K_t^{nr}. \quad (20) \end{aligned}$$

where the parameter η^k reflects the importance of lagged composition of capital supply relative to the steady-state composition, $\widetilde{K}_*^{nr,cbi}/\widetilde{K}_*^{nr,kb}$, in determining adjustment costs. The parameter χ^k scales linearly the magnitude of the adjustment cost term.

The investment decision for the economy's three types of cpiatl stock do not interact in any way and so we can think of the optimizing representative capital-owner as facing separate problems for each type of investment good.

In considering its non-residential investment decision, the representative capital owner, taking as given the rental rate on non-residential capital in the economy's two sectors, $R_t^{nr,cbi}$ and $R_t^{nr,kb}$, the price of "capital" goods P_t^{kb}, and the stochastic discount factor $\beta^j\frac{\Lambda_{t+j}^{cnn}/P_{t+j}^{cbi}}{\Lambda_t^{cnn}/P_t^{cbi}}$, chooses non-residential investment, $E_t^{nr}(k)$, the non-residential capital stock next period, $K_{t+1}^{nr}(k)$, and the non-residential capital stocks that it supplies to the economy's two sectors, $K_t^{nr,cbi}(k)$ and $K_t^{nr,kb}(k)$, to maximize the present discounted value of profits subject to the law of motion governing the evolution of capital (equation 10) and given the costs implied

by moving capital between the sectors (equation 20).[1] Specifically, the capital owner solves:

$$\max_{\{E_t^{nr}(k),K_{t+1}^{nr}(k),K_t^{nr,cbi}(k),K_t^{nr,kb}(k)U_t^{cbi}(k),U_t^{kb}(k)\}_{t=0}^{\infty}}$$

$$\mathcal{E}_0\sum_{t=0}^{\infty}\beta^t\frac{\Lambda_t^{cnn}}{P_t^{cbi}}\left\{R_t^{nr,cbi}U_t^{cbi}(k)K_t^{nr,cbi}(k)+R_t^{nr,kb}U_t^{kb}(k)K_t^{nr,kb}(k)-P_t^{kb}E_t^{nr}(k)\right.$$

$$\left.-\kappa\left(\frac{(U_t^{cbi}(k))^{1+\psi}-1}{1+\psi}\right)P_t^{kb}K_t^{nr,cbi}-\kappa\left(\frac{(U_t^{kb}(k))^{1+\psi}-1}{1+\psi}\right)P_t^{kb}K_t^{nr,kb}\right\}$$

subject to

$$K_{\tau+1}^{nr}(k)=(1-\delta^{nr})K_\tau^{nr}(k)+A_\tau^{nr}E_\tau^{nr}(k)$$

$$-\frac{100\cdot\chi^{nr}}{2}\left(\frac{E_\tau^{nr}(k)-\eta^{nr}E_{\tau-1}^{nr}(k)\Gamma_t^{y,kb}-(1-\eta^{nr})\widetilde{E}_*^{nr}Z_\tau^mZ_\tau^{kb}}{K_\tau^{nr}}\right)^2K_\tau^{nr}$$

$$K_\tau^{nr,cbi}(k)+K_\tau^{nr,kb}(k)=K_\tau^{nr}(k)$$

$$-\frac{100\cdot\chi^k}{2}\left(\frac{K_\tau^{nr,cbi}(k)}{K_\tau^{nr,kb}(k)}-\eta^k\frac{K_{\tau-1}^{nr,cbi}}{K_{\tau-1}^{nr,kb}}-(1-\eta^k)\frac{\widetilde{K}_*^{nr,cbi}}{\widetilde{K}_*^{nr,kb}}\right)^2\frac{K_\tau^{nr,kb}}{K_\tau^{nr,cbi}}\cdot K_\tau^{nr}.$$

for $\tau=0,1,...,\infty$. (21)

In considering its consumer durable investment decision, the representative capital owner, taking as given the rental rate on consumer durables R_t^{cd}, the price of "capital" goods P_t^{kb}, and the stochastic discount factor $\beta^j\frac{\Lambda_{t+j}^{cnn}/P_{t+j}^{cbi}}{\Lambda_t^{cnn}/P_t^{cbi}}$, chooses its investment in consumer durables, $E_t^{cd}(k)$ and its consumer durable capital stock $K_{t+1}^{cd}(k)$ to maximize the present discounted value of its profits subject to the consumer durables evolution process. Specifically, the capital owner solves:

$$\max_{\{E_t^{cd}(k),K_{t+1}^{cd}(k)\}_{t=0}^{\infty}\}}\mathcal{E}_0\sum_{t=0}^{\infty}\beta^t\frac{\Lambda_t^{cnn}}{P_t^{cbi}}\left\{R_t^{cd}K_t^{cd}(k)-P_t^{kb}E_t^{cd}(k)\right\}$$

subject to $K_{\tau+1}^{cd}(k)=(1-\delta^{cd})K_\tau^{cd}(k)+A_\tau^{cd}E_\tau^{cd}(k)$

$$-\frac{100\cdot\chi^{cd}}{2}\left(\frac{E_\tau^{cd}(k)-\eta^{cd}E_{\tau-1}^{cd}(k)\Gamma_\tau^{x,kb}-(1-\eta^{cd})\widetilde{E}_*^{cd}Z_\tau^mZ_\tau^k}{K_\tau^{cd}}\right)^2K_\tau^{cd}$$

for $\tau=0,1,...,\infty$. (22)

Finally, in its residential investment decision, the representative capital owner, taking as given the rental rate on residential capital R_t^r, the price of "consumption" goods P_t^{cbi}, and the stochastic discount factor $\beta^j\frac{\Lambda_{t+j}^{cnn}/P_{t+j}^{cbi}}{\Lambda_t^{cnn}/P_t^{cbi}}$, chooses its level of residential investment,

[1] The economy's capital stock is also ultimately owned by the households, so that the relevant discount factor in comparing nominal earnings and expenditures in period t with those in period $t+j$ is $\beta^j\frac{\Lambda_{t+j}^{cnn}/P_{t+j}^{cbi}}{\Lambda_t^{cnn}/P_t^{cbi}}$.

$E_t^r(k)$ and its residential capital stock $K_{t+1}^r(k)$ to maximize the present discounted value of its profits subject to the residential evolution process. Specifically, the capital owner solves:

$$\max_{\{E_t^r(k),K_{t+1}^r(k)\}_{t=0}^{\infty}\}} \mathcal{E}_0 \sum_{t=0}^{\infty} \beta^t \frac{\Lambda_t^{cnn}}{P_t^{cbi}} \left\{ R_t^r K_t^r(k) - P_t^{cbi} E_t^r(k) \right\}$$

subject to

$$K_{\tau+1}^r(k) = (1-\delta^r)K_\tau^r(k) + A_\tau^r E_\tau^r(k) - \frac{100 \cdot \chi^r}{2} \left(\frac{E_\tau^r(k) - \eta^r E_{\tau-1}^r(k)\Gamma_\tau^{x,cbi} - (1-\eta^r)\widetilde{E}_*^r Z_\tau^m (Z_\tau^{kb})^\alpha (Z_\tau^{cbi})^{1-\alpha}}{K_\tau^{cd}} \right)^2 K_\tau^{cd}$$

for $\tau = 0, 1, ..., \infty$. (23)

2.4 Households

Household's utility, which is defined over three components of goods and services consumption as well as leisure, is described by equation (13). Since each household supplies its own differentiated variety of labor to each sector, $L_t^s(i)$, it is able to set its wage $W_t^s(i)$. It does this taking into account the demand schedule for its labor that it faces from intermediate goods sector s and the adjustment costs that it encounters in altering its wage and the composition of its labor. Analogous to to the constraint faced by intermediate goods producers, wage-setting adjustment costs are introduced to the model through the assumption that the act of altering wages absorbs some of the household's income. The act of redircting labor from one sector to the other also absorbs some of the household's income. These adjustment costs are reflected in the household's budget constraint:

$$\begin{aligned} R_t^{-1} B_{t+1}(i) = B_t(i) &+ \sum_{s=cbi,kb} (1+\sigma^{w,s}) W_t^s(i) L_t^s(i) + \mathit{Profits}_t(i) + \mathit{Govt.\ Transfers}_t(i) \\ &+ \mathit{Foreign\ Transfers}_t(i) - P_t^{cbi} E_t^{cnn}(i) - R_t^{cd} K_t^{cd} - R_t^r K_t^r \\ &- \sum_{s=cbi,kb} \frac{100 \cdot \chi^{w,s}}{2} \left(\frac{W_t^s(j)}{W_{t-1}^s(j)} - \eta^{w,s} \Pi_{t-1}^{w,s} - (1-\eta^{w,s}) \Pi_*^w \right)^2 W_t^s L_t^s \\ &- \frac{100 \cdot \chi^l}{2} \left(\frac{L_*^{cbi}}{L_*^{cbi} + L_*^{kb}} \cdot W_t^{cbi} + \frac{L_*^{kb}}{L_*^{cbi} + L_*^{kb}} \cdot W_t^{kb} \right) \\ &\qquad \times \left(\frac{L_t^{cbi}(i)}{L_t^{kb}(i)} - \eta^l \frac{L_{t-1}^{cbi}}{L_{t-1}^{kb}} - (1-\eta^l) \frac{L_*^{cbi}}{L_*^{kb}} \right)^2 \frac{L_t^{kb}}{L_t^{cbi}}. \end{aligned} \tag{24}$$

The variable $B_t(i)$ is the state-contingent value, in terms of the numeraire, of household i's asset holdings at the beginning of period t. We assume that there exists a riskfree one-period

bond, which pays one unit of the numeraire in each state, and denote its yield—that is, the gross nominal interest rate between periods t and $t+1$—by $R_t \equiv \left(\beta \frac{\mathcal{E}_t\{\Lambda^{cnn}_{t+1}/P^{cbi}_{t+1}\}}{\Lambda^{cnn}_t/P^{cbi}_t}\right)^{-1}$. The parameter $\sigma^{w,s} = (\Theta^{w,s}_* - 1)^{-1}$ is a subsidy to labor that is set to ensure that the economy's level of steady-state labor (and consequently output) is Pareto optimal. The ith household receives profits from three sources from (i) its part-ownership of the optimizing capital-owner $\mathit{Profits}^k_t(i)$, (ii) its part-ownership of firms in the consumption goods sector $\mathit{Profits}^{cbi}_t(i)$, and (iii) its part-ownership of firms in the capital goods sector $\mathit{Profits}^{kb}_t(i)$. These profits—which are not household specific so that households treat as given and do not optimize with respect to—are defined as follows

$$\begin{aligned}
\mathit{Profits}^k_t(i) =& R^{cd}_t K^{cd}_t - P^{kb}_t E^{cd}_t + R^r_t K^r_t - P^{cbi}_t E^r_t + R^{nr,cbi}_t U^{cbi}_t K^{nr,cbi}_t + R^{nr,kb}_t U^{kb}_t K^{nr,kb}_t \\
& - P^{kb}_t E^{nr}_t - \kappa \left(\frac{(U^{cbi}_t)^{1+\psi} - 1}{1+\psi}\right) P^{kb}_t K^{nr,cbi}_t - \kappa \left(\frac{(U^{kb}_t)^{1+\psi} - 1}{1+\psi}\right) P^{kb}_t K^{nr,kb}_t \qquad (25) \\
\mathit{Profits}^{cbi}_t(i) =& (1+\sigma^{p,cbi}) P^{cbi}_t X^{cbi}_t - R^{nr,cbi}_t K^{u,nr,cbi}_t - W^{cbi}_t L^{cbi}_t - \sigma^{p,cbi} P^{cbi}_t X^{f,cbi}_t \\
& - \frac{100 \cdot \chi^{p,cbi}}{2} \left(\Pi^{p,cbi}_t - \eta^{p,cbi} \Pi^{p,cbi}_{t-1} - (1-\eta^{p,cbi}) \Pi^{p,cbi}_*\right)^2 P^{cbi}_t X^{cbi}_t \qquad (26) \\
\mathit{Profits}^{kb}_t(i) =& (1+\sigma^{p,kb}) P^{kb}_t X^{kb}_t - R^{nr,kb}_t K^{u,nr,kb}_t - W^{kb}_t L^{kb}_t - \sigma^{p,kb} P^{kb}_t X^{f,kb}_t \\
& - \frac{100 \cdot \chi^{p,kb}}{2} \left(\Pi^{p,kb}_t - \eta^{p,kb} \Pi^{p,kb}_{t-1} - (1-\eta^{p,kb}) \Pi^{p,kb}_*\right)^2 P^{kb}_t X^{kb}_t. \qquad (27)
\end{aligned}$$

Government transfers are set so as to raise enough revenue to pay the subsidy on labor and for the governments purchases of business and institutions sector output, that is:

$$\mathit{Govt.\ Transfers}_t(i) = -\sum_{s=cbi,kb} \sigma^{w,s} W^s_t L^s_t - \sum_{s=cbi,kb} P^s_t G^s_t. \qquad (28)$$

Foreign transfers are given by

$$\mathit{Foreign\ Transfers}_t(i) = -\sum_{s=cbi,kb} P^s_t N X^s_t. \qquad (29)$$

Turning, finally, to the two adjustment cost terms, the parameter $\eta^{w,s}$ reflects the importance of lagged wage inflation relative to steady-state inflation in determining adjustment costs and Π^w_* denotes the steady-state rate of wage inflation (which is equal across sectors). The parameter $\chi^{w,s}$ scales linearly the magnitude of the adjustment cost term; in a flexible price model $\chi^{w,s}$ wouble be equal to zero. The parameter η^l reflects the importance of lagged composition of labor supply relative to the steady-state composition, L^{cbi}_*/L^{kb}_*, in determining adjustment costs. The parameter χ^l scales linearly the magnitude of the adjustment cost term.

The household, taking as given the expected path of the gross nominal interest rate, R_t, the price of "consumption" goods, P_t^{cbi}, the rental rates on the consumer durables capital stock, R_t^{cd}, and the residential capital stock, R_t^r, the aggregate wage rate in each sector W_t^s, profits income, government and foreign transfers, and the initial bond stock $B_0(i)$, chooses its expenditure on consumer non-durable goods and non-housing services, $E_t^{cnn}(i)$, its stocks of consumer durable goods K_t^{cd} and residential capital K_t^r to rent, its wage in each sector $W_t^s(i)$ and labor supply $L_t^s(i)$ to maximize its utility subject to its budget constraint. Specifically, the household solves:

$$\max_{\left\{E_t^{cnn}(i),K_t^{cd}(i),K_t^r(i),\{W_t^s(i),L_t^s(i)\}_{s=cbi,kb},B_{t+1}(i)\right\}_{t=0}^{\infty}}$$

$$\mathcal{E}_0\sum_{t=0}^{\infty}\beta^t\Big\{\varsigma^{cnn}\Xi_t^{cnn}\ln(E_t^{cnn}(i)-h^{cnn}E_{t-1}^{cnn}(i))+\varsigma^{cd}\Xi_t^{cd}\ln(K_t^{cd}(i)-h^{cd}K_{t-1}^{cd}(i))$$
$$+\varsigma^r\Xi_t^r\ln(K_t^r(i)-h^rK_{t-1}^r(i))-\varsigma^l\Xi_t^l\frac{(L_t^{cbi}(i)+L_t^{kb}(i))^{1+\nu}}{1+\nu}\Big\}.$$

subject to

$$R_\tau^{-1}B_{\tau+1}(i)=B_\tau(i)+\sum_{s=cbi,kb}(1+\sigma^{w,s})W_\tau^s(i)L_\tau^s(i)+\textit{Profits}_\tau(i)+\textit{Govt. Transfers}_\tau(i)$$
$$+\textit{Foreign Transfers}_\tau(i)-P_\tau^{cbi}E_\tau^{cnn}(i)-R_\tau^{cd}K_\tau^{cd}-R_\tau^rK_\tau^r$$
$$-\sum_{s=cbi,kb}\frac{100\cdot\chi^{w,s}}{2}\left(\frac{W_\tau^s(j)}{W_{\tau-1}^s(j)}-\eta^{w,s}\Pi_{\tau-1}^{w,s}-(1-\eta^{p,s})\Pi_*^w\right)^2W_\tau^sL_\tau^s$$
$$-\frac{100\cdot\chi^l}{2}\left(\frac{L_*^{cbi}}{L_*^{cbi}+L_*^{kb}}\cdot W_\tau^{cbi}+\frac{L_*^{kb}}{L_*^{cbi}+L_*^{kb}}\cdot W_\tau^{kb}\right)$$
$$\times\left(\frac{L_\tau^{cbi}(i)}{L_\tau^{kb}(i)}-\eta^l\frac{L_{\tau-1}^{cbi}}{L_{\tau-1}^{kb}}-(1-\eta^l)\frac{L_*^{cbi}}{L_*^{kb}}\right)^2\frac{L_\tau^{kb}}{L_\tau^{cbi}}.$$

$$L_\tau^{cbi}(i)=\left(\frac{W_\tau^{cbi}(i)}{W_\tau^{cbi}}\right)^{-\Theta_\tau^{l,cbi}}L_\tau^{cbi},\ \text{and}\ \ L_\tau^{kb}(i)=\left(\frac{W_\tau^{kb}(i)}{W_\tau^{kb}}\right)^{-\Theta_\tau^{l,kb}}L_\tau^{kb},\quad\text{for }\tau=0,1,...,\infty.\quad(30)$$

2.5 Market Clearing

There are a number of market clearing conditions that must be satisfied in our model. Market clearing in the slow-growing "consumption" goods and fast-growing "capital" goods sectors requires that:

$$X_t^{cbi}=\int_0^1E_t^{cnn}(i)di+\int_0^1E_t^r(k)dk+G_t^{cbi}+NX_t^{cbi}\text{ and }X_t^{kb}=\int_0^1E_t^{cd}(k)dk+\int_0^1E_t^{nr}(k)dk+G_t^{kb}+NX_t^{kb}.\quad(31)$$

where G_t^{cbi}, NX_t^{cbi}, G_t^{kb}, and NX_t^{kb} follow exogenous processes. The market clearing conditions for the labor and non-residential capital supplied and demanded in sector s are given by

$$L_t^s(i) = \int_0^1 L_t^s(i,j)dj \text{ and } \int_0^1 U(k)_t^s K_t^{nr,s}(k)dk = \int_0^1 K_t^{u,nr,s}(j)dj \text{ for all } i \in [0,1] \text{ and for } s = cbi, kb. \tag{32}$$

The market clearing conditions for consumer durables and residential capital are:

$$\int_0^1 K_t^{cd}(k)dk = \int_0^1 K_t^{cd}(i)di \text{ and } \int_0^1 K_t^r(k)dk = \int_0^1 K_t^r(i)di. \tag{33}$$

2.6 Identities

The model also consists of the following identities:

$$W_t^s(i) = \Pi_t^{w,s}(i)W_{t-1}^s(i) \text{ and } W_t^s = \Pi_t^{w,s}W_{t-1}^s \text{ for all } i \in [0,1] \text{ and for } s = cbi, kb, \text{ and} \tag{34}$$

$$P_t^s(j) = \Pi_t^{p,s}(j)P_{t-1}^s(j) \text{ and } P_t^s = \Pi_t^{p,s}P_{t-1}^s \text{ for all } j \in [0,1] \text{ and for } s = cbi, kb. \tag{35}$$

2.7 Personal Consumption Expenditure and GDP

Four variables of considerable interest to policy-makers do not automatically appear in our model. These are the real growth rate of personal consumption expenditure and the inflation rate of its associated deflator, and real GDP growth and the inflation rate of its associated deflator. These latter two variables are especially important in the current versions of the model since they both appear in the monetary authority interest rate rule. At a latter date we will likely consider including the rate of increase in the PCE deflator in the policy rule, at which point our PCE aggregates will be of equal importance. At present, however, the PCE aggregates constructed below lead nowhere in the model.

The real growth rate of PCE, is defined in the model in the same way that is it in the NIPA, that is, as a chain-weighted aggregate of expenditure growth on consumer non-durable goods and non-housing services (E_t^{cnn}/E_{t-1}^{cnn}), consumer durable goods (E_t^{cd}/E_{t-1}^{cd}), and consumer housing services (E_t^{ch}/E_{t-1}^{ch}):

$$\begin{aligned} H_t^{pce} = &\left(E_t^{cnn}/E_{t-1}^{cnn}\right)^{\frac{1}{2}\cdot\frac{P_t^{cbi}E_t^{cnn}}{P_t^{cbi}E_t^{cnn}+P_t^{kb}E_t^{cd}+P_t^{ch}E_t^{ch}}+\frac{1}{2}\cdot\frac{P_{t-1}^{cbi}E_{t-1}^{cnn}}{P_{t-1}^{cbi}E_{t-1}^{cnn}+P_t^{kb}E_{t-1}^{cd}+P_{t-1}^{ch}E_{t-1}^{ch}}} \\ &\times\left(E_t^{cd}/E_{t-1}^{cd}\right)^{\frac{1}{2}\cdot\frac{P_t^{kb}E_t^{cd}}{P_t^{cbi}E_t^{cnn}+P_t^{kb}E_t^{cd}+P_t^{ch}E_t^{ch}}+\frac{1}{2}\cdot\frac{P_{t-1}^{kb}E_{t-1}^{cd}}{P_{t-1}^{cbi}E_{t-1}^{cnn}+P_t^{kb}E_{t-1}^{cd}+P_{t-1}^{ch}E_{t-1}^{ch}}} \\ &\times\left(E_t^{ch}/E_{t-1}^{ch}\right)^{\frac{1}{2}\cdot\frac{P_t^{ch}E_t^{ch}}{P_t^{cbi}E_t^{cnn}+P_t^{kb}E_t^{cd}+P_t^{ch}E_t^{ch}}+\frac{1}{2}\cdot\frac{P_{t-1}^{ch}E_{t-1}^{ch}}{P_{t-1}^{cbi}E_{t-1}^{cnn}+P_t^{kb}E_{t-1}^{cd}+P_{t-1}^{ch}E_{t-1}^{ch}}}, \end{aligned} \tag{36}$$

where E_t^{ch}, which denotes the service flow from the residential capital stock, is defined by:

$$E_t^{ch} = X_t^{ch} = \frac{\Gamma_*^{x,cbi} - \beta(1-\delta^r)}{\beta} \cdot K_t^r \tag{37}$$

The PCE inflation rate, $\Pi_t^{p,pce}$, is defined by:

$$\begin{aligned}\Pi_t^{p,pce} = &\left(P_t^{cbi}/P_{t-1}^{cbi}\right)^{\frac{1}{2}\cdot\frac{P_t^{cbi}E_t^{cnn}}{P_t^{cbi}E_t^{cnn}+P_t^{kb}E_t^{cd}+P_t^{ch}E_t^{ch}}+\frac{1}{2}\cdot\frac{P_{t-1}^{cbi}E_{t-1}^{cnn}}{P_{t-1}^{cbi}E_{t-1}^{cnn}+P_t^{kb}E_{t-1}^{cd}+P_{t-1}^{ch}E_{t-1}^{ch}}} \\ &\times\left(P_t^{kb}/P_{t-1}^{kb}\right)^{\frac{1}{2}\cdot\frac{P_t^{kb}E_t^{cd}}{P_t^{cbi}E_t^{cnn}+P_t^{kb}E_t^{cd}+P_t^{ch}E_t^{ch}}+\frac{1}{2}\cdot\frac{P_{t-1}^{kb}E_{t-1}^{cd}}{P_{t-1}^{cbi}E_{t-1}^{cnn}+P_t^{kb}E_{t-1}^{cd}+P_{t-1}^{ch}E_{t-1}^{ch}}} \\ &\times\left(P_t^{ch}/P_{t-1}^{ch}\right)^{\frac{1}{2}\cdot\frac{P_t^{ch}E_t^{ch}}{P_t^{ch}E_t^{cnn}+P_t^{kb}E_t^{cd}+P_t^{ch}E_t^{ch}}+\frac{1}{2}\cdot\frac{P_{t-1}^{ch}E_{t-1}^{ch}}{P_{t-1}^{cbi}E_{t-1}^{cnn}+P_t^{kb}E_{t-1}^{cd}+P_{t-1}^{ch}E_{t-1}^{ch}}}.\end{aligned} \tag{38}$$

where P_t^{ch}, which denotes the price of the service flow from the residential capital stock, is defined by:

$$P_t^{ch} = \frac{\beta}{\Gamma_*^{x,cbi} - \beta(1-\delta^r)} \cdot R_t^r. \tag{39}$$

Real GDP growth is also constructed in the model in the same way that it is in the NIPA, that is, as a chain-weighted aggregate of output growth in the slow-growing "consumption" goods sector (X_t^{cbi}/X_{t-1}^{cbi}), the fast-growing "capital" goods sector (X_t^{kb}/X_{t-1}^{kb}), household—or housing services—sector (X_t^{ch}/X_{t-1}^{ch}), and government—or government services—sector (X_t^{cg}/X_{t-1}^{cg}). Specifically,

$$\begin{aligned}H_t^{gdp} = &\left(X_t^{cbi}/X_{t-1}^{cbi}\right)^{\frac{1}{2}\cdot\frac{P_t^{cbi}X_t^{cbi}}{P_t^{cbi}X_t^{cbi}+P_t^{kb}X_t^{kb}+P_t^{ch}X_t^{ch}+P_t^{cg}X_t^{cg}}+\frac{1}{2}\cdot\frac{P_{t-1}^{cbi}X_{t-1}^{cbi}}{P_{t-1}^{cbi}X_{t-1}^{cbi}+P_{t-1}^{kb}X_{t-1}^{kb}+P_{t-1}^{ch}X_{t-1}^{ch}+P_{t-1}^{cg}X_{t-1}^{cg}}} \\ &\times\left(X_t^{kb}/X_{t-1}^{kb}\right)^{\frac{1}{2}\cdot\frac{P_t^{kb}X_t^{kb}}{P_t^{cbi}X_t^{cbi}+P_t^{kb}X_t^{kb}+P_t^{ch}X_t^{ch}+P_t^{cg}X_t^{cg}}+\frac{1}{2}\cdot\frac{P_{t-1}^{kb}X_{t-1}^{kb}}{P_{t-1}^{cbi}X_{t-1}^{cbi}+P_{t-1}^{kb}X_{t-1}^{kb}+P_{t-1}^{ch}X_{t-1}^{ch}+P_{t-1}^{cg}X_{t-1}^{cg}}} \\ &\times\left(X_t^{ch}/X_{t-1}^{ch}\right)^{\frac{1}{2}\cdot\frac{P_t^{ch}X_t^{ch}}{P_t^{cbi}X_t^{cbi}+P_t^{kb}X_t^{kb}+P_t^{ch}X_t^{ch}+P_t^{cg}X_t^{cg}}+\frac{1}{2}\cdot\frac{P_{t-1}^{ch}X_{t-1}^{ch}}{P_{t-1}^{cbi}X_{t-1}^{cbi}+P_{t-1}^{kb}X_{t-1}^{kb}+P_{t-1}^{ch}X_{t-1}^{ch}+P_{t-1}^{cg}X_{t-1}^{cg}}} \\ &\times\left(X_t^{cg}/X_{t-1}^{cg}\right)^{\frac{1}{2}\cdot\frac{P_t^{cg}X_t^{cg}}{P_t^{cbi}X_t^{cbi}+P_t^{kb}X_t^{kb}+P_t^{ch}X_t^{ch}+P_t^{cg}X_t^{cg}}+\frac{1}{2}\cdot\frac{P_{t-1}^{cg}X_{t-1}^{cg}}{P_{t-1}^{cbi}X_{t-1}^{cbi}+P_{t-1}^{kb}X_{t-1}^{kb}+P_{t-1}^{ch}X_{t-1}^{ch}+P_{t-1}^{cg}X_{t-1}^{cg}}},\end{aligned} \tag{40}$$

where the growth rate of government services $H_t^{cg} = X_t^{cg}/X_{t-1}^{cg}$ output was defined in

equation (8). The GDP inflation rate, $\Pi_t^{p,gdp}$, is defined by:

$$\begin{aligned}\Pi_t^{p,gdp} = &\left(P_t^{cbi}/P_{t-1}^{cbi}\right)^{\frac{1}{2}\cdot\frac{P_t^{cbi}X_t^{cbi}}{P_t^{cbi}X_t^{cbi}+P_t^{kb}X_t^{kb}+P_t^{ch}X_t^{ch}+P_t^{cg}X_t^{cg}}+\frac{1}{2}\cdot\frac{P_{t-1}^{cbi}X_{t-1}^{cbi}}{P_{t-1}^{cbi}X_{t-1}^{cbi}+P_{t-1}^{kb}X_{t-1}^{kb}+P_{t-1}^{ch}X_{t-1}^{ch}+P_{t-1}^{cg}X_{t-1}^{cg}}}\\ &\times\left(P_t^{kb}/P_{t-1}^{kb}\right)^{\frac{1}{2}\cdot\frac{P_t^{kb}X_t^{kb}}{P_t^{cbi}X_t^{cbi}+P_t^{kb}X_t^{kb}+P_t^{ch}X_t^{ch}+P_t^{cg}X_t^{cg}}+\frac{1}{2}\cdot\frac{P_{t-1}^{kb}X_{t-1}^{kb}}{P_{t-1}^{cbi}X_{t-1}^{cbi}+P_{t-1}^{kb}X_{t-1}^{kb}+P_{t-1}^{ch}X_{t-1}^{ch}+P_{t-1}^{cg}X_{t-1}^{cg}}}\\ &\times\left(P_t^{ch}/P_{t-1}^{ch}\right)^{\frac{1}{2}\cdot\frac{P_t^{ch}X_t^{ch}}{P_t^{cbi}X_t^{cbi}+P_t^{kb}X_t^{kb}+P_t^{ch}X_t^{ch}+P_t^{cg}X_t^{cg}}+\frac{1}{2}\cdot\frac{P_{t-1}^{ch}X_{t-1}^{ch}}{P_{t-1}^{cbi}X_{t-1}^{cbi}+P_{t-1}^{kb}X_{t-1}^{kb}+P_{t-1}^{ch}X_{t-1}^{ch}+P_{t-1}^{cg}X_{t-1}^{cg}}}\\ &\times\left(P_t^{cg}/P_{t-1}^{cg}\right)^{\frac{1}{2}\cdot\frac{P_t^{cg}X_t^{cg}}{P_t^{cbi}X_t^{cbi}+P_t^{kb}X_t^{kb}+P_t^{ch}X_t^{ch}+P_t^{cg}X_t^{cg}}+\frac{1}{2}\cdot\frac{P_{t-1}^{cg}X_{t-1}^{cg}}{P_{t-1}^{cbi}X_{t-1}^{cbi}+P_{t-1}^{kb}X_{t-1}^{kb}+P_{t-1}^{ch}X_{t-1}^{ch}+P_{t-1}^{cg}X_{t-1}^{cg}}},\end{aligned} \tag{41}$$

The growth rate of the price of government services $\Pi_t^{cg} = P_t^{cg}/P_{t-1}^{cg}$ is exogenous and assumed to follow AR(1) process. Specifically, letting $\pi_t^{cg} = \ln \Pi_t^{cg} - \ln \Pi_*^{cg}$, we allow

$$\pi_t^{cg} = \rho^{\pi,cg} \cdot \pi_{t-1}^{cg} + \epsilon_t^{\pi,cg}.$$

2.8 Monetary Authority

The central bank sets monetary policy in accordance with an Taylor-type interest-rate feedback rule. Policymakers smoothly adjust the actual interest rate R_t to its target level $\bar{R}_t$

$$R_t = (R_{t-1})^{\phi^r} \left(\bar{R}_t\right)^{1-\phi^r} \exp\left[\epsilon_t^r\right], \tag{42}$$

where the parameter ϕ^r reflects the degree of interest rate smoothing, while ϵ_t^r represents a monetary policy shock. The central bank's target nominal interest rate $\bar{R}_t$ is given by:

$$\bar{R}_t = \left(\Pi_t^{p,gdp}/\Pi_*^{p,gdp}\right)^{\phi^{\pi,gdp}} \left(\Delta\Pi_t^{p,gdp}\right)^{\phi^{\Delta\pi,gdp}} \left(H_t^{y,gdp}/H_*^{y,gdp}\right)^{\phi^{h,gdp}} \left(\Delta H_t^{y,gdp}\right)^{\phi^{\Delta h,gdp}} R_*. \tag{43}$$

where R_* denotes the economy's steady-state nominal interest rate (which is equal to $(1/\beta)\Pi_*^{p,c}\Gamma_*^{z,m}(\Gamma_*^{z,kb})^{\alpha}(\Gamma_*^{z,cbi})^{1-\alpha}$) and $\phi^{\pi,gdp}$, $\phi^{\Delta\pi,gdp}$, $\phi^{h,gdp}$, and $\phi^{\Delta h,gdp}$ denote the weights in the feedback rule.

2.9 Government and Foreign Sector

The government in our model:

- Pays production subsidies of $\sigma^{p,s}\int_0^1 P_t^s(j)X_t^s(j)dj$ to intermediate-good producing firms and labor subsidies of $\sigma^{w,s}\int_0^1 W_t^s(i)L_t^s(i)di$ to households to ensure that production and labor supply are at their Pareto optimal levels.

- Makes expenditure outlays of G_t^{cbi} and G_t^{kb} on the economy's two-final goods.
- Levies lump-sum taxes on intermediate-good producing firms and households to balance its budget in every period.

The foreign sector in our model pays lump-sum transfers equal to $-P_t^{cbi}NX_t^{cbi} - P_t^{kb}NX_t^{kb}$ to households.

Letting $E_t^{gf,cbi} = G_t^{cbi} + NX_t^{cbi}$ and $E_t^{gf,kb} = G_t^{kb} + NX_t^{kb}$ we can define the stationary version of each variable as $\widetilde{E}_t^{gf,cbi} = \frac{E_t^{gf,kb}}{Z_t^m (Z_t^{kb})^{\alpha} (Z_t^{cbi})^{1-\alpha}}$ and $\widetilde{E}_t^{gf,kb} = \frac{E_t^{gf,cbi}}{Z_t^m Z_t^{kb}}$. Letting $\widetilde{E}_*^{gf,cbi}$ and $\widetilde{E}_*^{gf,kb}$ denote the steady-state values of the stationary variables we can define:

$$e_t^{gf,cbi} = \rho^{gf,cbi} e_{t-1}^{gf,cbi} + \epsilon^{gf,cbi} \text{ and } e_t^{gf,kb} = \rho^{gf,kb} e_{t-1}^{gf,kb} + \epsilon^{gf,kb}.$$

where $e_t^{gf,cbi} = \ln \widetilde{E}_t^{gf,cbi} - \ln \widetilde{E}_*^{gf,cbi}$ and $e_t^{gf,kb} = \ln \widetilde{E}_t^{gf,kb} - \ln \widetilde{E}_*^{gf,kb}$.

3 Equilibrium

Before characterizing equilibrium in this model, we define three additional variables, the price of installed non-residential capital $Q_t^{nr}(k)$, the price of installed consumer durables capital $Q_t^{cd}(k)$, and the price of installed residential capital $Q_t^{r}(k)$. These variables are the lagrange multiplier on the capital evolution equations that would be implied by the kth capital owner's profit-maximization problems (described in equations 21, 22, and 23).

Equilibrium in our model is an allocation:

$$\Big\{ H_t^{gdp}, H_t^{pce}, X_t^{cbi}, \{X_t^{cbi}(j)\}_{j=0}^1, X_t^{kb}, \{X_t^{kb}(j)\}_{j=0}^1, X_t^{ch}, \{E_t^{nr}(k)\}_{k=0}^1, \{E_t^{r}(k)\}_{k=0}^1, \{E_t^{cd}(k)\}_{k=0}^1, \\ \{E_t^{cnn}(i)\}_{i=0}^1, E_t^{ch}, \{\Lambda_t^{r}(i)\}_{i=0}^1, \{\Lambda_t^{cd}(i)\}_{i=0}^1, \{\Lambda_t^{cnn}(i)\}_{i=0}^1, \{\Lambda_t^{l,cbi}(i)\}_{i=0}^1, \{\Lambda_t^{l,kb}(i)\}_{i=0}^1, \\ \{L_t^{cbi}(i)\}_{i=0}^1, \{\{L_t^{cbi}(i,j)\}_{i=0}^1\}_{j=0}^1, \{L_t^{kb}(i)\}_{i=0}^1, \{\{L_t^{kb}(i,j)\}_{i=0}^1\}_{j=0}^1, \{U_t^{cbi}(j)\}_{j=0}^1, \{U_t^{kb}(j)\}_{j=0}^1, \\ \{K_t^{u,nr,cbi}(j)\}_{j=0}^1, \{K_t^{u,nr,kb}(j)\}_{j=0}^1, \{K_t^{nr,cbi}(k)\}_{k=0}^1, \{K_t^{nr,kb}(k)\}_{k=0}^1, \{K_{t+1}^{nr}(k)\}_{k=0}^1, \\ \{K_{t+1}^{r}(k)\}_{k=0}^1, \{K_{t+1}^{r}(i)\}_{i=0}^1, \{K_{t+1}^{cd}(k)\}_{k=0}^1, \{K_{t+1}^{cd}(i)\}_{i=0}^1 \Big\}_{t=0}^{\infty}$$

and a sequence of values

$$\Big\{ \Pi_t^{p,gdp}, \Pi_t^{p,pce}, \Pi_t^{p,cbi}, \{\Pi_t^{p,cbi}(j)\}_{j=0}^1, \Pi_t^{p,kb}, \{\Pi_t^{p,kb}(j)\}_{j=0}^1, \Pi_t^{w,cbi}, \{\Pi_t^{w,cbi}(i)\}_{j=0}^1, \Pi_t^{w,kb}, \{\Pi_t^{w,kb}(i)\}_{j=0}^1, \\ \frac{P_t^{kb}}{P_t^{cbi}}, \left\{ \frac{P_t^{kb}(j)}{P_t^{cbi}} \right\}_{j=0}^1, \frac{W_t^{cbi}}{P_t^{cbi}}, \left\{ \frac{W_t^{cbi}(i)}{P_t^{cbi}} \right\}_{i=0}^1, \frac{W_t^{kb}}{P_t^{cbi}}, \left\{ \frac{W_t^{kb}(i)}{P_t^{cbi}} \right\}_{i=0}^1, \frac{R_t^{nr,cbi}}{P_t^{cbi}}, \frac{R_t^{nr,kb}}{P_t^{cbi}}, \frac{R_t^{nr}}{P_t^{cbi}}, \\ \frac{R_t^{r}}{P_t^{cbi}}, \frac{R_t^{cd}}{P_t^{cbi}}, \left\{ \frac{MC_t^{cbi}(j)}{P_t^{cbi}} \right\}_{j=0}^1, \left\{ \frac{MC_t^{kb}(j)}{P_t^{cbi}} \right\}_{j=0}^1, \left\{ \frac{Q_t^{nr}(k)}{P_t^{cbi}} \right\}_{k=0}^1, \left\{ \frac{Q_t^{r}(k)}{P_t^{cbi}} \right\}_{k=0}^1, \left\{ \frac{Q_t^{cd}(k)}{P_t^{cbi}} \right\}_{k=0}^1, R_t \Big\}_{t=0}^{\infty}$$

that satisfy the following conditions:

- The model's two representative final-good producing firms solve (15) for $s = cbi$ and kb;
- All intermediate-good producers $j \in [0,1]$ solve (16), (17), and (19) for $s = cbi$ and kb;
- All capital owners $k \in [0,1]$ solves (21), (22), and (23);
- All households $i \in [0,1]$ solve (30);
- The two final goods markets clear as in (31);
- All intermediate goods markets clear (by construction);
- The labor and non-residential capital markets clear as in (32);
- The consumer durable and residential capital rental markets clear as in (33);
- The identities given in (34) hold, but are modified slightly to $\frac{W_t^s(i)}{P_t^c} = \frac{\Pi_t^{w,s}(i)}{\Pi_t^{p,c}} \cdot \frac{W_{t-1}^s(i)}{P_{t-1}^c}$ and $\frac{W_t^s}{P_t^c} = \frac{\Pi_t^{w,s}}{\Pi_t^{p,c}} \cdot \frac{W_{t-1}^s}{P_{t-1}^c}$ for all $i \in [0,1]$ and for $s = cbi, kb$;
- The identities given in (35) hold, although are modified slightly to $\frac{P_t^s(j)}{P_t^c} = \frac{\Pi_t^{p,s}(j)}{\Pi_t^{p,c}} \cdot \frac{P_{t-1}^s(j)}{P_{t-1}^c}$ and $\frac{P_t^k}{P_t^c} = \frac{\Pi_t^{p,k}}{\Pi_t^{p,c}} \cdot \frac{P_{t-1}^k}{P_{t-1}^c}$ for all $j \in [0,1]$ and for $s = cbi, kb$;
- The monetary authority follows (42) and (43);
- The BEA constructs measures of the growth rate of PCE and GDP aggregates and price defined by equations (36), (38), (40), and (41), where imputed consumption and production of housing services and its price are defined by (37) and (39).

In solving these problems agents take as given the initial values of K_0^{nr}, K_0^r, K_0^{cd}, and R_{-1}, and the sequence of exogenous variables

$$\Big\{A_t^{cbi}, A_t^{kb}, A_t^m, \Gamma_t^{z,cbi}, \Gamma_t^{z,kb}, \Gamma_t^{z,m}, \Theta_t^{y,cbi}, \Theta_t^{y,kb}, \Theta_t^{l,cbi}, \Theta_t^{l,kb}, \\ A_t^{nr}, A_t^r, A_t^{cd}, \Xi_t^{cnn}, \Xi_t^{cd}, \Xi_t^r, \Xi_t^l, E_t^{gf,cbi}, E_t^{gf,kb}, H_t^{cg}, \Pi_t^{p,cg}\Big\}_{t=0}^{\infty}$$

implied by the sequence of shocks

$$\Big\{\epsilon_t^{a,cbi}, \epsilon_t^{a,kb}, \epsilon_t^{a,m}, \epsilon_t^{z,cbi}, \epsilon_t^{z,kb}, \epsilon_t^{z,m}, \epsilon_t^{\theta,y,cbi}, \epsilon_t^{\theta,y,kb}, \epsilon_t^{\theta,l,cbi}, \epsilon_t^{\theta,l,kb}, \\ \epsilon_t^{a,nr}, \epsilon_t^{a,r}, \epsilon_t^{a,cd}, \epsilon_t^{\xi,cnn}, \epsilon_t^{\xi,cd}, \epsilon_t^{\xi,r}, \epsilon_t^{\xi,l}, \epsilon_t^{gf,cbi}, \epsilon_t^{gf,kb}, \epsilon_t^{h,cg}, \epsilon_t^{\pi,cg}, \epsilon_t^r\Big\}_{t=0}^{\infty}.$$

We estimating the log-linearized, symmetric and stationary version of the model described above. Equilibrium in the symmetric and non-stationary version of the model is defined in appendix A of the paper, and equilibrium in the symmetric and stationary version of the model is defined in appendix B.

4 Data

The model is estimated using 11 data series. The series, each from the Bureau of Economic Analysis's National Income and Product Accounts except where noted, are:

1. Nominal gross domestic product.
2. Nominal consumption expenditure on nondurables and services excluding housing services.
3. Nominal consumption expenditure on durables.
4. Nominal residential investment expenditure.
5. Nominal business investment expenditure, which equals nominal gross private domestic investment minus nominal residential investment.
6. GDP price inflation.
7. Inflation for consumer nondurables and services (which represents inflation in the fast-growing "capital" goods sector).
8. Inflation for consumer durables (which represents inflation in the slow-growing "consumption" goods sector).
9. Hours, which equals hours of all persons in the non-farm business sector from the Bureau of Labor Statistics; we scale up this measure of hours by the ratio of nominal spending in our model to nominal non-farm business sector output in order to model a level of hours more appropriate for the total economy.
10. Wage inflation, which equals compensation per hour in the non-farm business sector from the Bureau of Labor Statistics.
11. The federal funds rate, from the Federal Reserve Board.

Chain-weighted price and quantity indices for each of the model's expenditure and output variables can be retrieved from the US NIPA. The construction of these price and quantitiy series are as follows:

Expenditures on consumer non-durable goods and non-housing services (*ECNN*) is the chain-weighted sum of Personal consumption expenditures on non-durable goods and Personal consumption expenditures on services (Table 1.1 of the NIPA) with Owner-occupied nonfarm dwellings and Tenant-occupied nonfarm dwellings (Table 2.4) chain-subtracted.

Expenditures on consumer housing services (*ECH*) is the chain-weighted sum of housing services from Owner-occupied nonfarm dwellings and Tenant-occupied nonfarm dwellings (Table 2.4).

Expenditures on consumer durable goods (*ECD*) is Personal consumption expenditures on durable goods (Table 1.1).

Expenditures on residential investment (*ER*) is Gross private domestic residential investment (Table 1.1).

Expenditures on non-residential investment (*ENR*) is the chain-weighted sum of Gross private domestic nonresidential investment and the change in private inventories (Table 1.1).

Government and foreign demand for the output produced by the slow-growing business and institutions sector (*EGFC*) can be thought of as the chain aggregation of the two demand components. Government demand is the chain-weighted sum of Government consumption expenditures (Table 3.9) with the Value added of general government (Table 3.10) chain-subtracted. Foreign demand is Exports of goods and services excluding Non-automotive capital goods, Automotive vehicles, engines, and parts, and Non-automotive consumer durables goods (Table 4.2) with Imports of goods and services excluding Non-automotive capital goods, Automotive vehicles, engines, and parts, and Non-automotive consumer durables goods (also Table 4.2) chain-subtracted out.

Government and foreign demand for the output produced by the fast-growing business sector (*EGFK*) can also be thought of as the chain aggregation of the two demand components. Government demand is Government gross investment (Table 3.9). Foreign demand is Exports of Non-automotive capital goods, Automotive vehicles, engines, and parts, and Non-automotive consumer durables goods (Table 4.2) with Imports of Non-automotive capital goods, Automotive vehicles, engines, and parts, and Non-automotive consumer durables goods (also Table 4.2) chain-subtracted out.

Production in the slow-growing part of the business and institutions sector (*XCBI*) is the chain-weighted sum of Expenditures on consumer non-durable goods and non-housing services (*ECNN*), Expenditures on residential investment (*ER*), and Government and foreign demand for the output produced by the slow-growing business and institutions sector (*EGFC*).

Production in the fast-growing part of the business sector (*XKB*) is the chain-weighted sum of Expenditures on consumer durable goods (*ECD*), Expenditures on non-residential investment (*ENR*), and Government and foreign demand for the output produced by the fast-growing business sector (*EGFK*).

Production in the slow-growing household sector (*XCH*) is equal to Expenditures on consumer housing services (*ECH*).

Production in the very slow-growing government sector (*XCG*) is equal to the Value added of general government (Table 3.10).

As noted above a price index can be constructed for every expenditure and output variable. Our theoretical model can, however, have only one price per output good. In the identities:

$$P^{XCBI}\ XCBI \;=\; P^{ECNN}\ ECNN \;+\; P^{ER}\ ER \;+\; P^{EGFC}\ EGFC, \tag{44}$$

$$P^{XKB}\ XKB \;=\; P^{ECD}\ ECD \;+\; P^{ENR}\ ENR \;+\; P^{EGFK}\ EGFK, \text{ and} \tag{45}$$

$$P^{XCH}\ XCH \;=\; P^{ECH}\ ECH, \tag{46}$$

that hold between the model's expenditure and output variables, only in the last equation does $P^{XCH} = P^{ECH}$ (which is by definition, since $XCH = ECH$). In the first two equations $P^{XCBI} \neq P^{ECNN} \neq P^{ER} \neq P^{EGFC}$ and $P^{XKB} \neq P^{ECD} \neq P^{ENR} \neq P^{EGFK}$.

To bring our data in line with our model the series must be modified slightly. Although the three price indices P^{XCBI}, P^{ECNN}, and P^{ER} are not identical they do not display any dramatic relative price swings. Similarly, the indicies P^{XKB}, P^{ECD}, and P^{ENR}, while not identical, do not exhibit any large swings. Consequently, I make the following modification to the data. I re-write equations (44) and (45) as

$$P^{ECNN}\left[\frac{P^{XCBI}XCBI}{P^{ECNN}}\right]$$
$$= P^{ECNN}ECNN \;+\; P^{ECNN}\left[\frac{P^{ER}ER}{P^{ECNN}}\right] \;+\; P^{ECNN}\left[\frac{P^{EGFC}EGFC}{P^{ECNN}}\right], \text{ and}$$
$$P^{ECD}\left[\frac{P^{XKB}XKB}{P^{ECD}}\right] = P^{ECD}ECD \;+\; P^{ECD}\left[\frac{P^{ENR}ENR}{P^{ECD}}\right] \;+\; P^{ECD}\left[\frac{P^{EGFK}EGFK}{P^{ECD}}\right],$$

and let $\widetilde{XCBI} = \dfrac{P^{XCBI}XCBI}{P^{ECNN}}$, $\widetilde{ER} = \dfrac{P^{ER}ER}{P^{ECNN}}$, $\widetilde{EGFC} = \dfrac{P^{EGFC}EGFC}{P^{ECNN}}$,

$\widetilde{XKB} = \dfrac{P^{XKB}XKB}{P^{ECD}}$, $\widetilde{ENR} = \dfrac{P^{ENR}ENR}{P^{ECD}}$, and $\widetilde{EGFK} = \dfrac{P^{EGFK}EGFK}{P^{ECD}}$.

Equations (44), (45), and (46) can then be written as:

$$P^{ECNN}\ \widetilde{XCBI} = P^{ECNN}\ ECNN + P^{ECNN}\ \widetilde{ER} + P^{ECNN}\ \widetilde{EGFC}, \tag{47}$$

$$P^{ECD}\ \widetilde{XKB} = P^{ECD}\ ECD + P^{ECD}\ \widetilde{ENR} + P^{ECD}\ \widetilde{EGFK}, \text{ and} \tag{48}$$

$$P^{ECH}\ XCH = P^{ECH}\ ECH, \tag{49}$$

The above renormalization of the data does mean that the series for real expenditures on residential investment ($\widetilde{ER}$) and for expenditures on non-residential investment ($\widetilde{ENR}$) are no longer the series published in the NIPA. The qualitative patterns in the data are unaltered by these normalizations, although the precise values of the series do change thereby resulting in a slight bias in the adjusted series. This divergence can be handled, if and when a focus on the behavior of these real series is desired, by the standard assumption that the data that we are using for residential investment and for nonresidential investment is measured with error.

Since the series for production in the slow-growing part of the business and institutions sector ($XCBI$) and production in the fast-growing part of the business sector (XKB) are also changed by the renormalization, the chain-aggregation of $\widetilde{XCBI}$, $\widetilde{XKB}$, XCH, and XCG—which we denote by $\widetilde{XGDP}$—is no longer equal to the NIPA series for true GDP ($XGDP$). In addition the GDP price deflator implied by the sectoral price indices is no longer equal to the true GDP price deflator. Specifically the true GDP price deflator—denoted P^{XGDP}—is the chain-weighted aggregation of P^{XCBI}, P^{XKB}, P^{XCH}, and P^{XCG}, the GDP price deflator implied by the sector price indices is the chain-weighted aggregation of $\widetilde{P}^{XCBI}(= P^{ECNN})$, $\widetilde{P}^{XBK}(= P^{ECD})$, P^{XCH}, and P^{XCG}, denoted by $\widetilde{P}^{XGDP}$. There are differences between the true and alternative measures even though their qualitative features are unchanged. Importantly, however, there is essentially no bias in these estimates. The divergence between the estimates is more evident for the GDP price deflator, although this partly reflects the fact that the magnitudes of our readings for GDP price inflation are smaller than the readings for GDP growth. Note also, that GDP price inflation is of secondary interest to PCE price inflation, which is not altered by the price normalizations performed in equations (47), (48), and (49).

5 The Empirical Model

5.1 Model Parameters

The model' calibrated parameters are presented in Tables 1, while the estimated parameters are presented in Table 2. We based out decision on which parameters to calibrate and which to estimate on how informative the data were likely to be on the parameter, as well as identification and overparameterization issues.

The first three columns of Table 2 outlines our assumptions about the prior distributions of the estimated parameters, the remaining columns describe the parameters' posterior distributions.

[To be completed]

5.2 Variance Decompositions

Table 3 presents forecast error variance decompositions at various (quarterly) horizons at the posterior mode of the parameter estimates for key variables and shocks.

[To be completed]

5.3 Impulse Responses

Figures 1 to 13 present the impulse responses of key variables to the models four preference shocks (Ξ_t^{cnn}, Ξ_t^{cd}, Ξ_t^{r}, and Ξ_t^{l}), four capital efficiency shocks (A_t^{nr}, A_t^{cd}, and A_t^{r}), the autonomous spending shock ($H^{x,gf}$), mark-up shocks ($\Theta_t^{x,cbi}$ and $\Theta_t^{x,cbi}$), technology shocks ($\Gamma_t^{x,m}$ and $\Gamma_t^{kb,m}$), and monetary policy shock (ϵ^r).

[To be completed]

5.4 Implied Paths

Figure 14 compares the one-step ahead DGE forecast to the actual observations for the data. Figures 15 and 16 show the 95 percent confidence banks and the median of the smoothed paths of the persistent structural shocks.

[To be completed]

A The Symmetric Equilibrium

The symmetric equilibrium is an allocation:

$$\Big\{H_t^{gdp}, H_t^{pce}, X_t^{cbi}, X_t^{kb}, X_t^{ch}, E_t^{nr}, E_t^{r}, E_t^{cd}, E_t^{cnn}, E_t^{ch}, \Lambda_t^{r}, \Lambda_t^{cd}, \Lambda_t^{cnn}, \Lambda_t^{l,cbi}, \Lambda_t^{l,kb},$$
$$L_t^{cbi}, L_t^{kb}, U_t^{cbi}, U_t^{kb}, K_t^{u,nr,cbi}, K_t^{u,nr,kb}, K_t^{nr,cbi}, K_t^{nr,kb}, K_{t+1}^{nr}, K_{t+1}^{r}, K_{t+1}^{cd}\Big\}_{t=0}^{\infty}$$

and a sequence of values

$$\Big\{\Pi_t^{p,gdp}, \Pi_t^{p,pce}, \Pi_t^{p,cbi}, \Pi_t^{p,kb}, \Pi_t^{p,ch}, \Pi_t^{w,cbi}, \Pi_t^{w,kb}, \frac{P_t^{kb}}{P_t^{cbi}}, \frac{P_t^{ch}}{P_t^{cbi}}, \frac{W_t^{cbi}}{P_t^{cbi}}, \frac{W_t^{kb}}{P_t^{cbi}},$$
$$\frac{R_t^{nr,cbi}}{P_t^{cbi}}, \frac{R_t^{nr,kb}}{P_t^{cbi}}, \frac{R_t^{nr}}{P_t^{cbi}}, \frac{R_t^{r}}{P_t^{cbi}}, \frac{R_t^{cd}}{P_t^{cbi}}, \frac{MC_t^{cbi}}{P_t^{cbi}}, \frac{MC_t^{kb}}{P_t^{cbi}}, \frac{Q_t^{nr}}{P_t^{cbi}}, \frac{Q_t^{r}}{P_t^{cbi}}, \frac{Q_t^{cd}}{P_t^{cbi}}, R_t\Big\}_{t=0}^{\infty}$$

that satisfy the symmetric versions of the first order conditions implied by:

- The intermediate-good producers' second cost-minimization problem (described in 17) and profit-maximization problem (described in 19);
- The capital owners' profit-maximization problems (described in 21, 22, and 23); and
- The households' utility maximization problems (described in 30);

and the symmetric versions of the model's other equilibrium conditions, specifically:

- The model's market clearing conditions of which only the following need to be explicitly included:

$$X_t^{cbi} = E_t^{cnn} + E_t^{r} + E_t^{gf,cbi},\ X_t^{kb} = E_t^{cd} + E_t^{nr} + E_t^{gf,kb},$$
$$K_t^{u,nr,cbi} = U_t^{cbi} K_t^{nr,cbi},\ \text{and}\ K_t^{u,nr,kb} = U_t^{kb} K_t^{nr,kb}.$$

- The identities between real wages, relative prices, and different wage and price inflation rates:

$$\frac{P_t^{kb}}{P_t^{cbi}} = \frac{\Pi_t^{p,kb}}{\Pi_t^{p,cbi}} \cdot \frac{P_{t-1}^{kb}}{P_{t-1}^{cbi}}, \frac{P_t^{ch}}{P_t^{cbi}} = \frac{\Pi_t^{p,ch}}{\Pi_t^{p,cbi}} \cdot \frac{P_{t-1}^{ch}}{P_{t-1}^{cbi}},\ \text{and}\ \frac{W_t^{s}}{P_t^{cbi}} = \frac{\Pi_t^{w,s}}{\Pi_t^{p,cbi}} \cdot \frac{W_{t-1}^{s}}{P_{t-1}^{cbi}}\ \ \text{for}\ s = cbi, kb$$

- Equations (42) and (43) that describe the behavior of monetary policy; and
- Equations (36), (37), (38), (39), (40), and (41) that define the growth rates of PCE and GDP aggregates and prices as well as the imputed consumption/production and price of housing services.

In solving these problems agents take as given the initial values of K_0^{nr}, K_0^r, K_0^{cd}, and R_{-1}, and the sequence of exogenous variables

$$\Big\{A_t^{cbi}, A_t^{kb}, A_t^m, \Gamma_t^{z,cbi}, \Gamma_t^{z,kb}, \Gamma_t^{z,m}, \Theta_t^{y,cbi}, \Theta_t^{y,kb}, \Theta_t^{l,cbi}, \Theta_t^{l,kb}, \\ A_t^{nr}, A_t^r, A_t^{cd}, \Xi_t^{cnn}, \Xi_t^{cd}, \Xi_t^r, \Xi_t^l, E_t^{gf,cbi}, E_t^{gf,kb}, H_t^{cg}, \Pi_t^{p,cg}\Big\}_{t=0}^{\infty}$$

implied by the sequence of shocks

$$\Big\{\epsilon_t^{a,cbi}, \epsilon_t^{a,kb}, \epsilon_t^{a,m}, \epsilon_t^{z,cbi}, \epsilon_t^{z,kb}, \epsilon_t^{z,m}, \epsilon_t^{\theta,y,cbi}, \epsilon_t^{\theta,y,kb}, \epsilon_t^{\theta,l,cbi}, \epsilon_t^{\theta,l,kb}, \\ \epsilon_t^{a,nr}, \epsilon_t^{a,r}, \epsilon_t^{a,cd}, \epsilon_t^{\xi,cnn}, \epsilon_t^{\xi,cd}, \epsilon_t^{\xi,r}, \epsilon_t^{\xi,l}, \epsilon_t^{gf,cbi}, \epsilon_t^{gf,kb}, \epsilon_t^{h,cg}, \epsilon_t^{\pi,cg}, \epsilon_t^r\Big\}_{t=0}^{\infty}.$$

Note that the first-order conditions implied by the **final good producing firms' cost minimization problem** (described in 15) and the **first step of the intermediate-good producers cost-minimization problem** (described in 16) hold but are redundant in the symmetric equilibrium.

The symmetric first-order conditions implied by the **second step of the intermediate-goods producing firms' cost minimization problems** (equation 17) are:

$$L_t^s = (1-\alpha)\cdot X_t^s \cdot \frac{MC_t^s}{W_t^s} \qquad \text{for } s = cbi, bk \tag{50}$$

$$K_t^{u,nr,s} = \alpha \cdot X_t^s \cdot \frac{MC_t^s}{R_t^{nr,s}} \qquad \text{for } s = cbi, kb. \tag{51}$$

$$X_t^s = \left(A_t^m Z_t^m A_t^s Z_t^s L_t^s\right)^{1-\alpha} \left(K_t^{u,nr,s}\right)^{\alpha} \qquad \text{for } s = cbi, kb. \tag{52}$$

The key equation from the **intermediate-goods producing firms' profit maximization problems** (equation 19) are the price Phillips curves

$$\begin{aligned}\Theta_t^{x,s} MC_t^s X_t^s &= (1+\sigma^{p,s})\left(\Theta_t^{x,s}-1\right) P_t^s X_t^s \\ &+ 100\cdot\chi^{p,s}\left(\Pi_t^{p,s} - \eta^{p,s}\Pi_{t-1}^{p,s} - (1-\eta^{p,s})\Pi_*^{p,s}\right)\Pi_t^{p,s} P_t^s X_t^s \\ &- \beta\mathcal{E}_t\left\{\frac{\Lambda_{t+1}^{cnn}/P_{t+1}^{cbi}}{\Lambda_t^{cnn}/P_t^{cbi}}\cdot 100\cdot\chi^{p,s}\left(\Pi_{t+1}^{p,s} - \eta^{p,s}\Pi_t^{p,s} - (1-\eta^{p,s})\Pi_*^{p,s}\right)\Pi_{t+1}^{p,s} P_{t+1}^s X_{t+1}^s\right\} \\ &\qquad \text{for } s = cbi, kb.\end{aligned} \tag{53}$$

The symmetric first-order conditions implied by the **non-residential part of the capital owners' profit-maximization problem** (equation 21) are:

$$Q_t^{nr} = \beta \mathcal{E}_t \left\{ \frac{\Lambda_{t+1}^{cnn}/P_{t+1}^{cbi}}{\Lambda_t^{cnn}/P_t^{cbi}} \left(R_{t+1}^{nr} + (1-\delta^{nr}) Q_{t+1}^{nr} \right) \right\} \tag{54}$$

$$R_t^{nr,cbi} = \frac{R_t^{nr}}{U_t^{cbi}} \left[1 + 100 \cdot \chi^k \left(\frac{K_t^{nr,cbi}}{K_t^{nr,kb}} - \eta^k \frac{K_{t-1}^{nr,cbi}}{K_{t-1}^{nr,kb}} - (1-\eta^k) \frac{\widetilde{K}_*^{nr,cbi}}{\widetilde{K}_*^{nr,kb}} \right) \frac{K_t^{nr}}{K_t^{nr,cbi}} \right] \tag{55}$$

$$R_t^{nr,kb} = \frac{R_t^{nr}}{U_t^{kb}} \left[1 - 100 \cdot \chi^k \left(\frac{K_t^{nr,cbi}}{K_t^{nr,kb}} - \eta^k \frac{K_{t-1}^{nr,cbi}}{K_{t-1}^{nr,kb}} - (1-\eta^k) \frac{\widetilde{K}_*^{nr,cbi}}{\widetilde{K}_*^{nr,kb}} \right) \frac{K_t^{nr}}{K_t^{nr,kb}} \right] \tag{56}$$

$$U_t^s = \left(\frac{1}{\kappa} \cdot \frac{R_t^{nr,s}}{P_t^{kb}} \right)^{\frac{1}{\psi}} \qquad s = cbi, kb \tag{57}$$

$$\begin{aligned} P_t^{kb} = {} & Q_t^{nr} \left[A_t^{nr} - 100 \cdot \chi^{nr} \left(\frac{E_t^{nr} - \eta^{nr} E_{t-1}^{nr} \Gamma_t^{x,kb} - (1-\eta^{nr}) \widetilde{E}_*^{nr} Z_t^m Z_t^k}{K_t^{nr}} \right) \right] \\ & + \beta \mathcal{E}_t \left\{ \frac{\Lambda_{t+1}^{cnn}/P_{t+1}^{cbi}}{\Lambda_t^{cnn}/P_t^{cbi}} \cdot Q_{t+1}^{nr} \cdot 100 \cdot \chi^{nr} \cdot \eta^{nr} \cdot \Gamma_{t+1}^{x,kb} \right. \\ & \left. \times \left(\frac{E_{t+1}^{nr} - \eta^{nr} E_t^{nr} \Gamma_{t+1}^{x,kb} - (1-\eta^{nr}) \widetilde{E}_*^{nr} Z_t^m Z_t^k}{K_{t+1}^{nr}} \right) \right\} \end{aligned} \tag{58}$$

$$K_{t+1}^{nr} = (1-\delta^{nr}) K_t^{nr} + A_t^{nr} E_t^{nr} - \frac{100 \cdot \chi^{nr}}{2} \left(\frac{E_t^{nr} - \eta^{nr} E_{t-1}^{nr} \Gamma_t^{x,kb} - (1-\eta^{nr}) \widetilde{E}_*^{nr} Z_t^m Z_t^k}{K_t^{nr}} \right)^2 K_t^{nr} \tag{59}$$

$$\begin{aligned} & K_t^{nr,cbi} + K_t^{nr,kb} \\ & \quad = K_t^{nr} - \frac{100 \cdot \chi^k}{2} \left(\frac{K_t^{nr,cbi}}{K_t^{nr,kb}} - \eta^k \frac{K_{t-1}^{nr,cbi}}{K_{t-1}^{nr,kb}} - (1-\eta^k) \frac{\widetilde{K}_*^{nr,cbi}}{\widetilde{K}_*^{nr,kb}} \right)^2 \frac{K_t^{nr,kb}}{K_t^{nr,cbi}} \cdot K_t^{nr}. \end{aligned} \tag{60}$$

The symmetric first-order conditions implied by the **consumer durables part of the capital owners' profit-maximization problem** (equation 22) are:

$$Q_t^{cd} = \beta \mathcal{E}_t \left\{ \frac{\Lambda_{t+1}^{cnn}/P_{t+1}^{cbi}}{\Lambda_t^{cnn}/P_t^{cbi}} \left(R_{t+1}^{cd} + (1-\delta^{cd}) Q_{t+1}^{cd} \right) \right\} \tag{61}$$

$$\begin{aligned} P_t^{kb} = Q_t^{cd} & \left[A_t^{cd} - 100 \cdot \chi^{cd} \left(\frac{E_t^{cd} - \eta^{cd} E_{t-1}^{cd} \Gamma_t^{x,kb} - (1-\eta^{cd}) \widetilde{E}_*^{cd} Z_t^m Z_t^k}{K_t^{cd}} \right) \right] \\ & + \beta \mathcal{E}_t \left\{ \frac{\Lambda_{t+1}^{cnn}/P_{t+1}^{cbi}}{\Lambda_t^{cnn}/P_t^{cbi}} \cdot Q_{t+1}^{cd} \cdot 100 \cdot \chi^{cd} \cdot \eta^{cd} \cdot \Gamma_{t+1}^{x,kb} \right. \\ & \qquad \left. \times \left(\frac{E_{t+1}^{cd} - \eta^{cd} E_t^{cd} \Gamma_{t+1}^{x,kb} - (1-\eta^{cd}) \widetilde{E}_*^{cd} Z_{t+1}^m Z_{t+1}^k}{K_{t+1}^{cd}} \right) \right\} \end{aligned} \tag{62}$$

$$K_{t+1}^{cd} = (1-\delta^{cd}) K_t^{cd} + A_t^{cd} E_t^{cd} - \frac{100 \cdot \chi^{cd}}{2} \left(\frac{E_t^{cd} - \eta^{cd} E_{t-1}^{cd} \Gamma_t^{x,kb} - (1-\eta^{cd}) \widetilde{E}_*^{cd} Z_t^m Z_t^k}{K_t^{cd}} \right)^2 K_t^{cd} \tag{63}$$

The symmetric first-order conditions implied by the **residential part of the capital owners' profit-maximization problem** (equation 23) are:

$$Q_t^r = \beta \mathcal{E}_t \left\{ \frac{\Lambda_{t+1}^{cnn}/P_{t+1}^{cbi}}{\Lambda_t^{cnn}/P_t^{cbi}} \left(R_{t+1}^r + (1-\delta^r) Q_{t+1}^r \right) \right\} \tag{64}$$

$$\begin{aligned} P_t^{cbi} = Q_t^r & \left[A_t^r - 100 \cdot \chi^r \left(\frac{E_t^r - \eta^r E_{t-1}^r \Gamma_t^{x,cbi} - (1-\eta^r) \widetilde{E}_*^r Z_t^m (Z_t^k)^\alpha (Z_t^c)^{1-\alpha}}{K_t^r} \right) \right] \\ & + \beta \mathcal{E}_t \left\{ \frac{\Lambda_{t+1}^{cnn}/P_{t+1}^{cbi}}{\Lambda_t^{cnn}/P_t^{cbi}} \cdot Q_{t+1}^r \cdot 100 \cdot \chi^r \cdot \eta^r \cdot \Gamma_{t+1}^{x,cbi} \right. \\ & \qquad \left. \times \left(\frac{E_{t+1}^r - \eta^r E_t^r \Gamma_{t+1}^{x,cbi} - (1-\eta^r) \widetilde{E}_*^r Z_{t+1}^m (Z_{t+1}^k)^\alpha (Z_{t+1}^c)^{1-\alpha}}{K_{t+1}^r} \right) \right\} \end{aligned} \tag{65}$$

$$K_{t+1}^r = (1-\delta^r) K_t^r + A_t^r E_t^r - \frac{100 \cdot \chi^r}{2} \left(\frac{E_t^r - \eta^r E_{t-1}^r \Gamma_t^{x,cbi} - (1-\eta^r) \widetilde{E}_*^r Z_t^m (Z_t^k)^\alpha (Z_t^c)^{1-\alpha}}{K_t^r} \right)^2 K_t^r \tag{66}$$

The symmetric (expenditure-related) first-order conditions implied by the **households' utility-maximization problem** are: (equation 30) are:

$$\frac{\Lambda_t^{cnn}}{P_t^{cbi}} = \beta R_t \mathcal{E}_t \left\{ \frac{\Lambda_{t+1}^{cnn}}{P_{t+1}^{cbi}} \right\} \tag{67}$$

$$\frac{\Lambda_t^{cnn}}{P_t^{cbi}} = \frac{\Lambda_t^{cd}}{R_t^{cd}} \tag{68}$$

$$\frac{\Lambda_t^{cnn}}{P_t^{cbi}} = \frac{\Lambda_t^{r}}{R_t^{r}} \tag{69}$$

$$\Lambda_t^{cnn} = \varsigma^{cnn} \cdot \frac{\Xi_t^{cnn}}{E_t^{cnn} - h^{cnn} E_{t-1}^{cnn}} - \beta \varsigma^{cnn} \mathcal{E}_t \left\{ \frac{h^{cnn} \Xi_{t+1}^{cnn}}{E_{t+1}^{cnn} - h^{cnn} E_t^{cnn}} \right\} \tag{70}$$

$$\Lambda_t^{cd} = \varsigma^{cd} \cdot \frac{\Xi_t^{cd}}{K_t^{cd} - h^{cd} K_{t-1}^{cd}} - \beta \varsigma^{cd} \mathcal{E}_t \left\{ \frac{h^{cd} \Xi_{t+1}^{cd}}{K_{t+1}^{cd} - h^{cd} K_t^{cd}} \right\} \tag{71}$$

$$\Lambda_t^{r} = \varsigma^{r} \cdot \frac{\Xi_t^{r}}{K_t^{r} - h^{r} K_{t-1}^{r}} - \beta \varsigma^{r} \mathcal{E}_t \left\{ \frac{h^{r} \Xi_{t+1}^{r}}{K_{t+1}^{r} - h^{r} K_t^{r}} \right\}. \tag{72}$$

The key equations from the households' labor-supply decision are the wage Phillips curves

$$\begin{aligned}
&\Theta_t^{l,cbi} \cdot \frac{\Lambda_t^{l,cbi}}{\Lambda_t^{cnn}} \cdot P_t^{cbi} L_t^{cbi} \\
&= \left(1 + \sigma^{w,cbi}\right) \left(\Theta_t^{l,cbi} - 1\right) W_t^{cbi} L_t^{cbi} \\
&- \Theta_t^{l,cbi} \cdot 100 \cdot \chi^{l} \left(\frac{L_*^{cbi}}{L_*^{cbi} + L_*^{cbi}} \cdot W_t^{cbi} + \frac{L_*^{kb}}{L_*^{cbi} + L_*^{cbi}} \cdot W_t^{kb} \right) \left(\frac{L_t^{cbi}}{L_t^{kb}} - \eta^{l} \frac{L_{t-1}^{cbi}}{L_{t-1}^{kb}} - (1-\eta^{l}) \frac{L_*^{cbi}}{L_*^{kb}} \right) \\
&+ 100 \cdot \chi^{w,cbi} \left(\Pi_t^{w,cbi} - \eta^{w,cbi} \Pi_{t-1}^{w,cbi} - (1-\eta^{w,cbi}) \Pi_*^{w,cbi} \right) \Pi_t^{w,cbi} W_t^{cbi} L_t^{cbi} \\
&- \beta \mathcal{E}_t \left\{ \frac{\Lambda_{t+1}^{cnn} / P_{t+1}^{cbi}}{\Lambda_t^{cnn} / P_t^{cbi}} \right. \\
&\qquad \left. \times 100 \cdot \chi^{w,cbi} \left(\Pi_{t+1}^{w,cbi} - \eta^{w,cbi} \Pi_t^{w,cbi} - (1-\eta^{w,cbi}) \Pi_*^{w,cbi} \right) \Pi_{t+1}^{w,cbi} W_{t+1}^{cbi} L_{t+1}^{cbi} \right\}
\end{aligned} \tag{73}$$

$$\begin{aligned}
&\Theta_t^{l,kb} \cdot \frac{\Lambda_t^{l,kb}}{\Lambda_t^{cnn}} \cdot P_t^{cbi} L_t^{kb} \\
&= \left(1 + \sigma^{w,kb}\right) \left(\Theta_t^{l,kb} - 1\right) W_t^{kb} L_t^{kb} \\
&+ \Theta_t^{l,kb} \cdot 100 \cdot \chi^{l} \left(\frac{L_*^{cbi}}{L_*^{cbi} + L_*^{cbi}} \cdot W_t^{cbi} + \frac{L_*^{kb}}{L_*^{cbi} + L_*^{cbi}} \cdot W_t^{kb} \right) \left(\frac{L_t^{cbi}}{L_t^{kb}} - \eta^{l} \frac{L_{t-1}^{cbi}}{L_{t-1}^{kb}} - (1-\eta^{l}) \frac{L_*^{cbi}}{L_*^{kb}} \right) \\
&+ 100 \cdot \chi^{w,kb} \left(\Pi_t^{w,kb} - \eta^{w,kb} \Pi_{t-1}^{w,kb} - (1-\eta^{w,kb}) \Pi_*^{w,kb} \right) \Pi_t^{w,kb} W_t^{kb} L_t^{kb} \\
&- \beta \mathcal{E}_t \left\{ \frac{\Lambda_{t+1}^{cnn} / P_{t+1}^{cbi}}{\Lambda_t^{cnn} / P_t^{cbi}} \right. \\
&\qquad \left. \times 100 \cdot \chi^{w,kb} \left(\Pi_{t+1}^{w,kb} - \eta^{w,kb} \Pi_t^{w,kb} - (1-\eta^{w,kb}) \Pi_*^{w,kb} \right) \Pi_{t+1}^{w,kb} W_{t+1}^{kb} L_{t+1}^{kb} \right\}
\end{aligned} \tag{74}$$

where

$$\Lambda_t^{l,cbi} = \Lambda_t^{l,kb} = \varsigma^l \Xi_t^l \left(L_t^{cbi} + L_t^{kb} \right)^{\nu} \tag{75}$$

B Equilibrium in the Symmetric and Stationary Model

The symmetric equilibrium is an allocation:

$$\Big\{ H_t^{gdp}, H_t^{pce}, \widetilde{X}_t^{cbi}, \widetilde{X}_t^{kb}, \widetilde{X}_t^{ch}, \widetilde{E}_t^{nr}, \widetilde{E}_t^{r}, \widetilde{E}_t^{cd}, \widetilde{E}_t^{cnn}, \widetilde{E}_t^{ch}, \widetilde{\Lambda}_t^{r}, \widetilde{\Lambda}_t^{cd}, \widetilde{\Lambda}_t^{cnn}, \Lambda_t^{l,cbi}, \Lambda_t^{l,kb},$$
$$L_t^{cbi}, L_t^{kb}, U_t^{cbi}, U_t^{kb}, \widetilde{K}_t^{u,nr,cbi}, \widetilde{K}_t^{u,nr,kb}, \widetilde{K}_t^{nr,cbi}, \widetilde{K}_t^{nr,kb}, \widetilde{K}_{t+1}^{nr}, \widetilde{K}_{t+1}^{r}, \widetilde{K}_{t+1}^{cd} \Big\}_{t=0}^{\infty}$$

and a sequence of values

$$\Big\{ \Pi_t^{p,gdp}, \Pi_t^{p,pce}, \Pi_t^{p,cbi}, \Pi_t^{p,kb}, \Pi_t^{p,ch}, \Pi_t^{w,cbi}, \Pi_t^{w,kb}, \widetilde{P}_t^{kb}, \widetilde{P}_t^{ch}, \widetilde{W}_t^{cbi}, \widetilde{W}_t^{kb},$$
$$\widetilde{R}_t^{nr,cbi}, \widetilde{R}_t^{nr,kb}, \widetilde{R}_t^{nr}, \widetilde{R}_t^{r}, \widetilde{R}_t^{cd}, \widetilde{MC}_t^{cbi}, \widetilde{MC}_t^{kb}, \widetilde{Q}_t^{nr}, \widetilde{Q}_t^{r}, \widetilde{Q}_t^{cd}, R_t \Big\}_{t=0}^{\infty}$$

that satisfy the stationary versions of the equations given in the preceding section, taking as given the initial values of K_0^{nr}, K_0^{r}, K_0^{cd}, and R_{-1}, and the sequence of exogenous variables

$$\Big\{ A_t^{cbi}, A_t^{kb}, A_t^{m}, \Gamma_t^{z,cbi}, \Gamma_t^{z,kb}, \Gamma_t^{z,m}, \Theta_t^{y,cbi}, \Theta_t^{y,kb}, \Theta_t^{l,cbi}, \Theta_t^{l,kb},$$
$$A_t^{nr}, A_t^{r}, A_t^{cd}, \Xi_t^{cnn}, \Xi_t^{cd}, \Xi_t^{r}, \Xi_t^{l}, E_t^{gf,cbi}, E_t^{gf,kb}, H_t^{cg}, \Pi_t^{p,cg} \Big\}_{t=0}^{\infty}$$

implied by the sequence of shocks

$$\Big\{ \epsilon_t^{a,cbi}, \epsilon_t^{a,kb}, \epsilon_t^{a,m}, \epsilon_t^{z,cbi}, \epsilon_t^{z,kb}, \epsilon_t^{z,m}, \epsilon_t^{\theta,y,cbi}, \epsilon_t^{\theta,y,kb}, \epsilon_t^{\theta,l,cbi}, \epsilon_t^{\theta,l,kb},$$
$$\epsilon_t^{a,nr}, \epsilon_t^{a,r}, \epsilon_t^{a,cd}, \epsilon_t^{\xi,cnn}, \epsilon_t^{\xi,cd}, \epsilon_t^{\xi,r}, \epsilon_t^{\xi,l}, \epsilon_t^{gf,cbi}, \epsilon_t^{gf,kb}, \epsilon_t^{h,cg}, \epsilon_t^{\pi,cg}, \epsilon_t^{r} \Big\}_{t=0}^{\infty}.$$

These equations are presented below.

The symmetric and stationary first-order conditions implied by the **second step of the intermediate-goods producing firms' cost minimization problems** (equation 17) are:

$$L_t^s = (1-\alpha) \cdot \widetilde{X}_t^s \cdot \frac{\widetilde{MC}_t^s}{\widetilde{W}_t^s} \qquad \text{for } s = cbi, bk \tag{76}$$

$$\frac{\widetilde{K}_t^{u,nr,s}}{\Gamma_t^{x,kb}} = \alpha \cdot \widetilde{X}_t^s \cdot \frac{\widetilde{MC}_t^s}{\widetilde{R}_t^{nr,s}} \qquad \text{for } s = cbi, kb. \tag{77}$$

$$\widetilde{X}_t^s = \left(A_t^m Z_t^m A_t^s Z_t^s L_t^s \right)^{1-\alpha} \left(\widetilde{K}_t^{u,nr,s} \right)^{\alpha} \qquad \text{for } s = cbi, kb. \tag{78}$$

The stationary price Phillips curves that are implied by the **intermediate-goods producing firms' profit maximization problems** (equation 19) are

$$\begin{aligned}\Theta_t^{x,cbi}\widetilde{MC}_t^{cbi}\widetilde{X}_t^{cbi} = & \left(1+\sigma^{p,cbi}\right)\left(\Theta_t^{x,cbi}-1\right)\widetilde{X}_t^{cbi} \\ & + 100\cdot\chi^{p,cbi}\left(\Pi_t^{p,cbi}-\eta^{p,cbi}\Pi_{t-1}^{p,cbi}-(1-\eta^{p,cbi})\Pi_*^{p,cbi}\right)\Pi_t^{p,cbi}\widetilde{X}_t^{cbi} \\ & - \beta\mathcal{E}_t\left\{\frac{\widetilde{\Lambda}_{t+1}^{cnn}}{\widetilde{\Lambda}_t^{cnn}}\cdot 100\cdot\chi^{p,cbi}\left(\Pi_{t+1}^{p,cbi}-\eta^{p,cbi}\Pi_t^{p,cbi}-(1-\eta^{p,cbi})\Pi_*^{p,cbi}\right)\Pi_{t+1}^{p,cbi}\widetilde{X}_{t+1}^{cbi}\right\}\end{aligned} \tag{79}$$

$$\begin{aligned}\Theta_t^{x,kb}\widetilde{MC}_t^{kb}\widetilde{X}_t^{kb} = & \left(1+\sigma^{p,kb}\right)\left(\Theta_t^{x,kb}-1\right)\widetilde{P}_t^{kb}\widetilde{X}_t^{kb} \\ & + 100\cdot\chi^{p,kb}\left(\Pi_t^{p,kb}-\eta^{p,kb}\Pi_{t-1}^{p,kb}-(1-\eta^{p,kb})\Pi_*^{p,kb}\right)\Pi_t^{p,kb}\widetilde{P}_t^{kb}\widetilde{X}_t^{kb} \\ & - \beta\mathcal{E}_t\left\{\frac{\widetilde{\Lambda}_{t+1}^{cnn}}{\widetilde{\Lambda}_t^{cnn}}\cdot 100\cdot\chi^{p,kb}\left(\Pi_{t+1}^{p,kb}-\eta^{p,kb}\Pi_t^{p,kb}-(1-\eta^{p,kb})\Pi_*^{p,kb}\right)\Pi_{t+1}^{p,kb}\widetilde{P}_{t+1}^{kb}\widetilde{X}_{t+1}^{kb}\right\}\end{aligned} \tag{80}$$

The symmetric and stationary first-order conditions implied by the **non-residential part**

of the capital owners' profit-maximization problem (equation 21) are:

$$\widetilde{Q}_t^{nr} = \beta \mathcal{E}_t \left\{ \frac{\widetilde{\Lambda}_{t+1}^{cnn}}{\widetilde{\Lambda}_t^{cnn}} \cdot \frac{1}{\Gamma_{t+1}^{x,kb}} \left(\widetilde{R}_{t+1}^{nr} + (1-\delta^{nr}) \widetilde{Q}_{t+1}^{nr} \right) \right\} \tag{81}$$

$$\widetilde{R}_t^{nr,cbi} = \frac{\widetilde{R}_t^{nr}}{U_t^{cbi}} \left[1 + 100 \cdot \chi^k \left(\frac{\widetilde{K}_t^{nr,cbi}}{\widetilde{K}_t^{nr,kb}} - \eta^k \frac{\widetilde{K}_{t-1}^{nr,cbi}}{\widetilde{K}_{t-1}^{nr,kb}} - (1-\eta^k) \frac{\widetilde{K}_*^{nr,cbi}}{\widetilde{K}_*^{nr,kb}} \right) \frac{\widetilde{K}_t^{nr}}{\widetilde{K}_t^{nr,cbi}} \right] \tag{82}$$

$$\widetilde{R}_t^{nr,kb} = \frac{\widetilde{R}_t^{nr}}{U_t^{kb}} \left[1 - 100 \cdot \chi^k \left(\frac{\widetilde{K}_t^{nr,cbi}}{\widetilde{K}_t^{nr,kb}} - \eta^k \frac{\widetilde{K}_{t-1}^{nr,cbi}}{\widetilde{K}_{t-1}^{nr,kb}} - (1-\eta^k) \frac{\widetilde{K}_*^{nr,cbi}}{\widetilde{K}_*^{nr,kb}} \right) \frac{\widetilde{K}_t^{nr}}{\widetilde{K}_t^{nr,kb}} \right] \tag{83}$$

$$U_t^s = \left(\frac{1}{\kappa} \cdot \frac{\widetilde{R}_t^{nr,s}}{\widetilde{P}_t^{kb}} \right)^{\frac{1}{\psi}} \qquad s = cbi, kb \tag{84}$$

$$\begin{aligned} \widetilde{P}_t^{kb} = {} & \widetilde{Q}_t^{nr} \left[A_t^{nr} - 100 \cdot \chi^{nr} \left(\frac{\widetilde{E}_t^{nr} - \eta^{nr} \widetilde{E}_{t-1}^{nr} - (1-\eta^{nr}) \widetilde{E}_*^{nr}}{\widetilde{K}_t^{nr}} \cdot \Gamma_t^{x,kb} \right) \right] \\ & + \beta \mathcal{E}_t \left\{ \frac{\widetilde{\Lambda}_{t+1}^{cnn}}{\widetilde{\Lambda}_t^{cnn}} \cdot \widetilde{Q}_{t+1}^{nr} \cdot 100 \cdot \chi^{nr} \cdot \eta^{nr} \left(\frac{\widetilde{E}_{t+1}^{nr} - \eta^{nr} \widetilde{E}_t^{nr} - (1-\eta^{nr}) \widetilde{E}_*^{nr}}{\widetilde{K}_{t+1}^{nr}} \cdot \Gamma_{t+1}^{x,kb} \right) \right\} \end{aligned} \tag{85}$$

$$\widetilde{K}_{t+1}^{nr} = (1-\delta^{nr}) \frac{\widetilde{K}_t^{nr}}{\Gamma_t^{x,kb}} + A_t^{nr} \widetilde{E}_t^{nr} - \frac{100 \cdot \chi^{nr}}{2} \left(\frac{\widetilde{E}_t^{nr} - \eta^{nr} \widetilde{E}_{t-1}^{nr} - (1-\eta^{nr}) \widetilde{E}_*^{nr}}{\widetilde{K}_t^{nr}} \cdot \Gamma_t^{x,kb} \right)^2 \frac{\widetilde{K}_t^{nr}}{\Gamma_t^{x,kb}} \tag{86}$$

$$\begin{aligned} & \widetilde{K}_t^{nr,cbi} + \widetilde{K}_t^{nr,kb} \\ & \quad = \widetilde{K}_t^{nr} - \frac{100 \cdot \chi^k}{2} \left(\frac{\widetilde{K}_t^{nr,cbi}}{\widetilde{K}_t^{nr,kb}} - \eta^k \frac{\widetilde{K}_{t-1}^{nr,cbi}}{\widetilde{K}_{t-1}^{nr,kb}} - (1-\eta^k) \frac{\widetilde{K}_*^{nr,cbi}}{\widetilde{K}_*^{nr,kb}} \right)^2 \frac{\widetilde{K}_t^{nr,kb}}{\widetilde{K}_t^{nr,cbi}} \cdot \widetilde{K}_t^{nr} \end{aligned} \tag{87}$$

The symmetric and stationary first-order conditions implied by the **consumer durables part of the capital owners'profit-maximization problem** (equation 22) are:

$$\widetilde{Q}_t^{cd} = \beta \mathcal{E}_t \left\{ \frac{\widetilde{\Lambda}_{t+1}^{cnn}}{\widetilde{\Lambda}_t^{cnn}} \cdot \frac{1}{\Gamma_{t+1}^{x,kb}} \left(\widetilde{R}_{t+1}^{cd} + (1-\delta^{cd}) \widetilde{Q}_{t+1}^{cd} \right) \right\} \tag{88}$$

$$\begin{aligned} \widetilde{P}_t^{kb} = {} & \widetilde{Q}_t^{cd} \left[A_t^{cd} - 100 \cdot \chi^{cd} \left(\frac{\widetilde{E}_t^{cd} - \eta^{cd} \widetilde{E}_{t-1}^{cd} - (1-\eta^{cd}) \widetilde{E}_*^{cd}}{\widetilde{K}_t^{cd}} \cdot \Gamma_t^{x,kb} \right) \right] \\ & + \beta \mathcal{E}_t \left\{ \frac{\widetilde{\Lambda}_{t+1}^{cnn}}{\widetilde{\Lambda}_t^{cnn}} \cdot \widetilde{Q}_{t+1}^{cd} \cdot 100 \cdot \chi^{cd} \cdot \eta^{cd} \left(\frac{\widetilde{E}_{t+1}^{cd} - \eta^{cd} \widetilde{E}_t^{cd} - (1-\eta^{cd}) \widetilde{E}_*^{cd}}{\widetilde{K}_{t+1}^{cd}} \cdot \Gamma_{t+1}^{x,kb} \right) \right\} \end{aligned} \tag{89}$$

$$\widetilde{K}_{t+1}^{cd} = (1-\delta^{cd}) \frac{\widetilde{K}_t^{cd}}{\Gamma_t^{x,kb}} + A_t^{cd} \widetilde{E}_t^{cd} - \frac{100 \cdot \chi^{cd}}{2} \left(\frac{\widetilde{E}_t^{cd} - \eta^{cd} \widetilde{E}_{t-1}^{cd} - (1-\eta^{cd}) \widetilde{E}_*^{cd}}{\widetilde{K}_t^{cd}} \cdot \Gamma_t^{x,kb} \right)^2 \frac{\widetilde{K}_t^{cd}}{\Gamma_t^{x,kb}}. \tag{90}$$

The symmetric and stationary first-order conditions implied by the **residential part of**

the capital owners' profit-maximization problem (equation 23) are:

$$\widetilde{Q}_t^r = \beta \mathcal{E}_t \left\{ \frac{\widetilde{\Lambda}_{t+1}^{cnn}}{\widetilde{\Lambda}_t^{cnn}} \cdot \frac{1}{\Gamma_{t+1}^{x,cbi}} \left(\widetilde{R}_{t+1}^r + (1-\delta^r)\widetilde{Q}_{t+1}^r \right) \right\} \tag{91}$$

$$\begin{aligned} 1 = \widetilde{Q}_t^r \left[A_t^r - 100 \cdot \chi^r \left(\frac{\widetilde{E}_t^r - \eta^r \widetilde{E}_{t-1}^r - (1-\eta^r)\widetilde{E}_*^r}{\widetilde{K}_t^r} \cdot \Gamma_t^{x,cbi} \right) \right] \\ + \beta \mathcal{E}_t \left\{ \frac{\widetilde{\Lambda}_{t+1}^{cnn}}{\widetilde{\Lambda}_t^{cnn}} \cdot \widetilde{Q}_{t+1}^r \cdot 100 \cdot \chi^r \cdot \eta^r \left(\frac{\widetilde{E}_{t+1}^r - \eta^r \widetilde{E}_t^r - (1-\eta^r)\widetilde{E}_*^r}{\widetilde{K}_{t+1}^r} \cdot \Gamma_{t+1}^{x,cbi} \right) \right\} \end{aligned} \tag{92}$$

$$\widetilde{K}_{t+1}^r = (1-\delta^r)\frac{\widetilde{K}_t^r}{\Gamma_t^{x,cbi}} + A_t^r \widetilde{E}_t^r - \frac{100 \cdot \chi^r}{2} \left(\frac{\widetilde{E}_t^r - \eta^r \widetilde{E}_{t-1}^r - (1-\eta^r)\widetilde{E}_*^r}{\widetilde{K}_t^r} \cdot \Gamma_t^{x,cbi} \right)^2 \frac{\widetilde{K}_t^r}{\Gamma_t^{x,cbi}}. \tag{93}$$

The symmetric and stationary (expenditure-related) first-order conditions implied by the **households' utility-maximization problem** are: (equation 30) are:

$$\widetilde{\Lambda}_t^{cnn} = \beta R_t \mathcal{E}_t \cdot \left\{ \widetilde{\Lambda}_t^{cnn} \cdot \frac{1}{\Pi_{t+1}^{cbi} \Gamma_{t+1}^{x,cbi}} \right\} \tag{94}$$

$$\widetilde{\Lambda}_t^{cnn} = \widetilde{\Lambda}_t^{cd} \cdot \frac{1}{\widetilde{R}_t^{cd}} \tag{95}$$

$$\widetilde{\Lambda}_t^{cnn} = \widetilde{\Lambda}_t^{r} \cdot \frac{1}{\widetilde{R}_t^{r}} \tag{96}$$

$$\widetilde{\Lambda}_t^{cnn} = \varsigma^{cnn} \cdot \frac{\Xi_t^{cnn}}{\widetilde{E}_t^{cnn} - (h^{cnn}/\Gamma_t^{x,cbi})\ \widetilde{E}_{t-1}^{cnn}} - \beta \varsigma^{cnn} \mathcal{E}_t \left\{ \frac{(h^{cnn}/\Gamma_{t+1}^{x,cbi})\ \Xi_{t+1}^{cnn}}{\widetilde{E}_{t+1}^{cnn} - (h^{cnn}/\Gamma_{t+1}^{x,cbi})\ \widetilde{E}_t^{cnn}} \right\} \tag{97}$$

$$\widetilde{\Lambda}_t^{cd} = \varsigma^{cd} \cdot \frac{\Xi_t^{cd}}{\widetilde{K}_t^{cd} - (h^{cd}/\Gamma_{t+1}^{x,kb})\ \widetilde{K}_{t-1}^{cd}} - \beta \varsigma^{cd} \mathcal{E}_t \left\{ \frac{(h^{cd}/\Gamma_{t+1}^{x,kb})\ \Xi_{t+1}^{cd}}{\widetilde{K}_{t+1}^{cd} - (h^{cd}/\Gamma_{t+1}^{x,kb})\ \widetilde{K}_t^{cd}} \right\} \tag{98}$$

$$\widetilde{\Lambda}_t^{r} = \varsigma^{r} \cdot \frac{\Xi_t^{r}}{\widetilde{K}_t^{r} - (h^{r}/\Gamma_t^{x,cbi})\ \widetilde{K}_{t-1}^{r}} - \beta \varsigma^{r} \mathcal{E}_t \left\{ \frac{(h^{r}/\Gamma_{t+1}^{x,cbi})\ \Xi_{t+1}^{r}}{\widetilde{K}_{t+1}^{r} - (h^{r}/\Gamma_{t+1}^{x,cbi})\ \widetilde{K}_t^{r}} \right\} \tag{99}$$

The key equations from the households' labor-supply decision are the wage Phillips curves

$$
\begin{aligned}
&\Theta_t^{l,cbi} \cdot \frac{\Lambda_t^{l,cbi}}{\widetilde{\Lambda}_t^{cnn}} \cdot L_t^{cbi} \\
&= \left(1+\sigma^{w,cbi}\right)\left(\Theta_t^{l,cbi}-1\right)\widetilde{W}_t^{cbi} L_t^{cbi} \\
&-\Theta_t^{l,cbi} \cdot 100 \cdot \chi^l \left(\frac{L_*^{cbi}}{L_*^{cbi}+L_*^{cbi}} \cdot \widetilde{W}_t^{cbi} + \frac{L_*^{kb}}{L_*^{cbi}+L_*^{cbi}} \cdot \widetilde{W}_t^{kb}\right)\left(\frac{L_t^{cbi}}{L_t^{kb}} - \eta^l \frac{L_{t-1}^{cbi}}{L_{t-1}^{kb}} - (1-\eta^l)\frac{L_*^{cbi}}{L_*^{kb}}\right) \\
&+100 \cdot \chi^{w,cbi}\left(\Pi_t^{w,cbi} - \eta^{w,cbi}\Pi_{t-1}^{w,cbi} - (1-\eta^{w,cbi})\Pi_*^{w,cbi}\right)\Pi_t^{w,cbi}\widetilde{W}_t^{cbi} L_t^{cbi} \\
&-\beta \mathcal{E}_t \left\{\frac{\widetilde{\Lambda}_{t+1}^{cnn}}{\widetilde{\Lambda}_t^{cnn}} \cdot 100 \cdot \chi^{w,cbi}\left(\Pi_{t+1}^{w,cbi} - \eta^{w,cbi}\Pi_t^{w,cbi} - (1-\eta^{w,cbi})\Pi_*^{w,cbi}\right)\Pi_{t+1}^{w,cbi}\widetilde{W}_{t+1}^{cbi} L_{t+1}^{cbi}\right\} \quad (100)
\end{aligned}
$$

$$
\begin{aligned}
&\Theta_t^{l,kb} \cdot \frac{\Lambda_t^{l,kb}}{\widetilde{\Lambda}_t^{cnn}} \cdot L_t^{kb} \\
&= \left(1+\sigma^{w,kb}\right)\left(\Theta_t^{l,kb}-1\right)\widetilde{W}_t^{kb} L_t^{kb} \\
&+\Theta_t^{l,kb} \cdot 100 \cdot \chi^l \left(\frac{L_*^{cbi}}{L_*^{cbi}+L_*^{cbi}} \cdot \widetilde{W}_t^{cbi} + \frac{L_*^{kb}}{L_*^{cbi}+L_*^{cbi}} \cdot \widetilde{W}_t^{kb}\right)\left(\frac{L_t^{cbi}}{L_t^{kb}} - \eta^l \frac{L_{t-1}^{cbi}}{L_{t-1}^{kb}} - (1-\eta^l)\frac{L_*^{cbi}}{L_*^{kb}}\right) \\
&+100 \cdot \chi^{w,kb}\left(\Pi_t^{w,kb} - \eta^{w,kb}\Pi_{t-1}^{w,kb} - (1-\eta^{w,kb})\Pi_*^{w,kb}\right)\Pi_t^{w,kb}\widetilde{W}_t^{kb} L_t^{kb} \\
&-\beta \mathcal{E}_t \left\{\frac{\widetilde{\Lambda}_{t+1}^{cnn}}{\widetilde{\Lambda}_t^{cnn}} \cdot 100 \cdot \chi^{w,kb}\left(\Pi_{t+1}^{w,kb} - \eta^{w,kb}\Pi_t^{w,kb} - (1-\eta^{w,kb})\Pi_*^{w,kb}\right)\Pi_{t+1}^{w,kb}\widetilde{W}_{t+1}^{kb} L_{t+1}^{kb}\right\} \quad (101)
\end{aligned}
$$

The model's other conditions for equilibrium, listed in appendix A for the non-stationary model, are transformed as follows in the stationary model:

- The model's market clearing conditions become:

$$
\widetilde{X}_t^{cbi} = \widetilde{E}_t^{cnn} + \widetilde{E}_t^{r} + \widetilde{E}_t^{gf,cbi}, \ \widetilde{X}_t^{kb} = \widetilde{E}_t^{cd} + \widetilde{E}_t^{nr} + \widetilde{E}_t^{gf,kb},
$$
$$
\widetilde{K}_t^{u,nr,cbi} = U_t^{cbi}\widetilde{K}_t^{nr,cbi}, \text{ and } \widetilde{K}_t^{u,nr,kb} = U_t^{kb}\widetilde{K}_t^{nr,kb}.
$$

- The identities between real wages, relative prices, and different wage and price inflation rates become:

$$
\widetilde{P}_t^{kb} = \frac{\Pi_t^{p,kb}}{\Pi_t^{p,cbi}} \cdot \frac{\Gamma_t^{x,kb}}{\Gamma_t^{x,cbi}} \cdot \widetilde{P}_{t-1}^{kb}, \ \widetilde{P}_t^{ch} = \frac{\Pi_t^{p,ch}}{\Pi_t^{p,cbi}} \cdot \widetilde{P}_{t-1}^{kb} \text{ and } \widetilde{W}_t^{s} = \frac{\Pi_t^{w,s}}{\Pi_t^{p,cbi}} \cdot \frac{1}{\Gamma_t^{x,cbi}} \cdot \widetilde{W}_{t-1}^{s} \text{ for } s = cbi, kb
$$

- Equations (42) and (43) that describe the behavior of monetary policy are already described in terms of stationary variables;
- The equations that define the growth rates of PCE and GDP prices are still given by (38) and (41) in the stationary model. The equations that define the growth rate of

aggregate PCE and GDP (previously 36 and 40) and the equations that describe the imputed consumption/production and price of housing services (previously 37 and 39) are now given by:

$$H_t^{pce} = \left(\frac{\Gamma_t^{x,cbi} \cdot \widetilde{E}_t^{cnn}}{\widetilde{E}_{t-1}^{cnn}} \right)^{\frac{1}{2} \cdot \frac{P_t^{cbi} E_t^{cnn}}{P_t^{cbi} E_t^{cnn} + P_t^{kb} E_t^{cd} + P_t^{ch} E_t^{ch}} + \frac{1}{2} \cdot \frac{P_{t-1}^{cbi} E_{t-1}^{cnn}}{P_{t-1}^{cbi} E_{t-1}^{cnn} + P_t^{kb} E_{t-1}^{cd} + P_{t-1}^{ch} E_{t-1}^{ch}}}$$

$$\times \left(\frac{\Gamma_t^{x,kb} \cdot \widetilde{E}_t^{cd}}{\widetilde{E}_{t-1}^{cd}} \right)^{\frac{1}{2} \cdot \frac{P_t^{kb} E_t^{cd}}{P_t^{cbi} E_t^{cnn} + P_t^{kb} E_t^{cd} + P_t^{ch} E_t^{ch}} + \frac{1}{2} \cdot \frac{P_{t-1}^{kb} E_{t-1}^{cd}}{P_{t-1}^{cbi} E_{t-1}^{cnn} + P_t^{kb} E_{t-1}^{cd} + P_{t-1}^{ch} E_{t-1}^{ch}}}$$

$$\times \left(\frac{\Gamma_t^{x,cbi} \cdot \widetilde{E}_t^{ch}}{\widetilde{E}_{t-1}^{ch}} \right)^{\frac{1}{2} \cdot \frac{P_t^{ch} E_t^{ch}}{P_t^{cbi} E_t^{cnn} + P_t^{kb} E_t^{cd} + P_t^{ch} E_t^{ch}} + \frac{1}{2} \cdot \frac{P_{t-1}^{ch} E_{t-1}^{ch}}{P_{t-1}^{cbi} E_{t-1}^{cnn} + P_t^{kb} E_{t-1}^{cd} + P_{t-1}^{ch} E_{t-1}^{ch}}}, \tag{102}$$

$$\widetilde{E}_t^{ch} = \widetilde{X}_t^{ch} = \frac{\Gamma_*^{x,cbi} - \beta(1-\delta^r)}{\beta} \cdot \widetilde{K}_t^r \tag{103}$$

$$\widetilde{P}_t^{ch} = \frac{\beta}{\Gamma_*^{x,cbi} - \beta(1-\delta^r)} \cdot \widetilde{R}_t^r. \tag{104}$$

$$H_t^{gdp}$$

$$= \left(\frac{\Gamma_t^{x,cbi} \cdot \widetilde{X}_t^{cbi}}{\widetilde{X}_{t-1}^{cbi}} \right)^{\frac{1}{2} \cdot \frac{P_t^{cbi} X_t^{cbi}}{P_t^{cbi} X_t^{cbi} + P_t^{kb} X_t^{kb} + P_t^{ch} X_t^{ch} + P_t^{cg} X_t^{cg}} + \frac{1}{2} \cdot \frac{P_{t-1}^{cbi} X_{t-1}^{cbi}}{P_{t-1}^{cbi} X_{t-1}^{cbi} + P_{t-1}^{kb} X_{t-1}^{kb} + P_{t-1}^{ch} X_{t-1}^{ch} + P_{t-1}^{cg} X_{t-1}^{cg}}}$$

$$\times \left(\frac{\Gamma_t^{x,kb} \cdot \widetilde{X}_t^{kb}}{\widetilde{X}_{t-1}^{kb}} \right)^{\frac{1}{2} \cdot \frac{P_t^{kb} X_t^{kb}}{P_t^{cbi} X_t^{cbi} + P_t^{kb} X_t^{kb} + P_t^{ch} X_t^{ch} + P_t^{cg} X_t^{cg}} + \frac{1}{2} \cdot \frac{P_{t-1}^{kb} X_{t-1}^{kb}}{P_{t-1}^{cbi} X_{t-1}^{cbi} + P_{t-1}^{kb} X_{t-1}^{kb} + P_{t-1}^{ch} X_{t-1}^{ch} + P_{t-1}^{cg} X_{t-1}^{cg}}}$$

$$\times \left(\frac{\Gamma_t^{x,cbi} \cdot \widetilde{X}_t^{ch}}{\widetilde{X}_{t-1}^{ch}} \right)^{\frac{1}{2} \cdot \frac{P_t^{ch} X_t^{ch}}{P_t^{cbi} X_t^{cbi} + P_t^{kb} X_t^{kb} + P_t^{ch} X_t^{ch} + P_t^{cg} X_t^{cg}} + \frac{1}{2} \cdot \frac{P_{t-1}^{ch} X_{t-1}^{ch}}{P_{t-1}^{cbi} X_{t-1}^{cbi} + P_{t-1}^{kb} X_{t-1}^{kb} + P_{t-1}^{ch} X_{t-1}^{ch} + P_{t-1}^{cg} X_{t-1}^{cg}}}$$

$$\times (H_t^{cg})^{\frac{1}{2} \frac{P_t^{cg} X_t^{cg}}{P_t^{cbi} X_t^{cbi} + P_t^{kb} X_t^{kb} + P_t^{ch} X_t^{ch} + P_t^{cg} X_t^{cg}} + \frac{1}{2} \cdot \frac{P_{t-1}^{cg} X_{t-1}^{cg}}{P_{t-1}^{cbi} X_{t-1}^{cbi} + P_{t-1}^{kb} X_{t-1}^{kb} + P_{t-1}^{ch} X_{t-1}^{ch} + P_{t-1}^{cg} X_{t-1}^{cg}}}, \tag{105}$$

C List of Model Parameters

h^{cnn} = Habit-persistence parameter for the consumption of non-durable goods and non-housing services.

h^{cd} = Habit-persistence parameter for the consumption of durable goods.

h^r = Habit-persistence parameter for the consumption of housing services.

α = The elasticity of output with respect to capital.

β = The household's discount factor.

δ^{cd} = The quarterly depreciation rate of consumer durables.

δ^{nr} = The quarterly depreciation rate of non-residential capital.

δ^{r} = The quarterly depreciation rate of residential capital.

η^{cd} = Parameter reflecting the relative importance of lagged investment spending in the consumer durables adjustment cost function.

η^{nr} = Parameter reflecting the relative importance of lagged investment spending in the non-residential capital adjustment cost function.

η^{r} = Parameter reflecting the relative importance of lagged investment spending in the residential capital adjustment cost function.

η^{k} = Parameter reflecting the relative importance of the lagged sectoral mix of non-residential capital in the non-residential capital sectoral adjustment cost function.

η^{l} = Parameter reflecting the relative importance of the lagged sectoral mix of labor in the labor sectoral adjustment cost function.

$\eta^{p,cbi}$ = Parameter reflecting the relative importance of lagged price inflation in the adjustment cost function for slow-growing "consumption" goods' prices.

$\eta^{p,kb}$ = Parameter reflecting the relative importance of lagged price inflation in the adjustment cost function for fast-growing "capital" goods' prices.

$\eta^{w,cbi}$ = Parameter reflecting the relative importance of lagged wage inflation in the adjustment cost function for wages in the slow-growing "consumption" goods sector.

$\eta^{w,kb}$ = Parameter reflecting the relative importance of lagged wage inflation in the adjustment cost function for wages in the fast-growing "capital" goods sector.

ν = Inverse labor supply elasticity.

$\rho^{a,cbi}$ = Persistence parameter in the AR(1) process describing the evolution of A_t^{cbi}.

$\rho^{a,kb}$ = Persistence parameter in the AR(1) process describing the evolution of A_t^{kb}.

$\rho^{a,m}$ = Persistence parameter in the AR(1) process describing the evolution of A_t^{m}.

$\rho^{a,nr}$ = Persistence parameter in the AR(1) process describing the evolution of A_t^{nr}.

$\rho^{a,cd}$ = Persistence parameter in the AR(1) process describing the evolution of A_t^{cd}.

$\rho^{a,r}$ = Persistence parameter in the AR(1) process describing the evolution of A_t^{r}.

$\rho^{z,cbi}$ = Persistence parameter in the AR(1) process describing the evolution of $\Gamma_t^{z,cbi}$.

$\rho^{z,kb}$ = Persistence parameter in the AR(1) process describing the evolution of $\Gamma_t^{z,kb}$.

$\rho^{z,m}$ = Persistence parameter in the AR(1) process describing the evolution of $\Gamma_t^{z,m}$.

$\rho^{\theta,x,cbi}$ = Persistence parameter in the AR(1) process describing the evolution of $\Theta_t^{x,cbi}$.

$\rho^{\theta,x,kb}$ = Persistence parameter in the AR(1) process describing the evolution of $\Theta_t^{x,kb}$.

$\rho^{\theta,l,cbi}$ = Persistence parameter in the AR(1) process describing the evolution of $\Theta_t^{l,cbi}$.

$\rho^{\theta,l,kb}$ = Persistence parameter in the AR(1) process describing the evolution of $\Theta_t^{l,kn}$.

$\rho^{\xi,cnn}$ = Persistence parameter in the AR(1) process describing the evolution of Ξ_t^{cnn}.

$\rho^{\xi,cd}$ = Persistence parameter in the AR(1) process describing the evolution of Ξ_t^{cd}.

$\rho^{\xi,r}$ = Persistence parameter in the AR(1) process describing the evolution of Ξ_t^{r}.

$\rho^{\xi,l}$ = Persistence parameter in the AR(1) process describing the evolution of Ξ_t^{l}.

ς^{cnn} = Co-efficient on the consumer non-durable goods and non-housing serives component of the utility function.

ς^{cd} = Co-efficient on the consumer durable goods component of the utility function.

ς^{r} = Co-efficient on the consumer housing serives component of the utility function.

ς^{l} = Co-efficient on the labor supply components of the utility function.

$\sigma^{p,cbi} = (\Theta_*^{p,cbi} - 1)^{-1}$ A subsidy to production in the slow-growing "consumption" goods sector set to ensure that the economy's level of steady-state output is optimal.

$\sigma^{p,kb} = (\Theta_*^{p,kb} - 1)^{-1}$ A subsidy to production in the fast-growing "capital" goods sector set to ensure that the economy's level of steady-state output is optimal.

$\sigma^{w,cbi} = (\Theta_*^{w,cbi} - 1)^{-1}$ A subsidy to labor supplied to the slow-growing "consumption" goods sector set to ensure that the economy's level of steady-state output is optimal.

$\sigma^{w,kb} = (\Theta_*^{w,kb} - 1)^{-1}$ A subsidy to labor supplied to the fast-growing "capital" goods sector set to ensure that the economy's level of steady-state output is optimal.

$\phi^{h,gdp}$ = Co-efficient on GDP growth in the monetary policy reaction function.

$\phi^{\Delta h,gdp}$ = Co-efficient on change in GDP growth in the monetary policy reaction function.

$\phi^{\pi,gdp}$ = Co-efficient on GDP price inflation in the monetary policy reaction function.

$\phi^{\Delta\pi,gdp}$ = Co-efficient on the change in GDP price inflation in the monetary policy reaction function.

ϕ^{r} = Co-efficient on lagged nominal interest rates in the monetary policy reaction function.

χ^{cd} = Investment adjustment costs in the consumer durables evolution equation.

χ^{nr} = Investment adjustment costs in the non-residential capital evolution equation.

χ^{r} = Investment adjustment costs in the residential capital evolution equation.

χ^{k} = Parameter reflecting the size of adjustment costs in the non-residential capital sectoral adjustment cost function.

χ^l = Parameter reflecting the size of adjustment costs in the labor sectoral adjustment cost function.

$\chi^{p,cbi}$ = Parameter reflecting the size of adjustment costs in re-setting prices in the slow-growing "consumption" goods sector.

$\chi^{p,kb}$ = Parameter reflecting the size of adjustment costs in re-setting prices in the fast-growing "capital" goods sector.

$\chi^{w,cbi}$ = Parameter reflecting the size of adjustment costs in re-setting wages for labor supplied to the slow-growing "consumption" goods sector.

$\chi^{w,kb}$ = Parameter reflecting the size of adjustment costs in re-setting wages for labor supplied to the fast-growing "capital" goods sector.

ψ = Elasticity of utilization costs.

D List of Endogenous and Exogenous Model Variables

A_t^{cbi} =

A_t^{kb} =

A_t^{m} =

A_t^{nr} =

A_t^{r} =

A_t^{cd} =

E_t^{nr} = Expenditures on goods in the fast-growing "capital" goods sector for use in non-residential investment.

E_t^{r} = Expenditures on goods in the slow-growing "consumption" goods sector for use in residential investment.

E_t^{cd} = Expenditures on goods in the fast-growing "capital" goods sector for use in consumer durables investment.

E_t^{cnn} = Expenditures on goods in the fast-growing "capital" goods sector for use in consumer non-durable goods and non-housing services.

E_t^{ch} = Expenditures on consumer housing services.

$E_t^{gf,cbi}$ = Exogenous expendiutre (by the government and foreign sector) on goods in the slow-growing "consumption" goods sector.

$E_t^{gf,kb}$ = Exogenous expendiutre (by the government and foreign sector) on goods in the

fast-growing "capital" goods sector.

H_t^{gdp} = Growth rate of real (chain-weighted) GDP.

H_t^{pce} = Growth rate of real (chain-weighted) PCE.

H_t^{cg} = Growth rate of real government output.

$K_t^{u,nr,cbi}$ = The amount of *utilized* non-residential capital used in the slow-growing "consumption"goods sector.

$K_t^{u,nr,kb}$ = The amount of *utilized* non-residential capital used in the fast-growing "capital" goods sector.

$K_t^{nr,cbi}$ = The *physical* amount of non-residential capital used in the slow-growing "consumption" goods sector.

$K_t^{nr,kb}$ = The *physical* amount of non-residential capital used in the fast-growing "capital" goods sector.

K_t^{nr} = The aggregate non-residential capital stock.

K_{t+1}^{r} = The residential capital stock.

K_{t+1}^{cd} = The consumer durables capital stock.

L_t^{cbi} = Labor used in the slow-growing "consumption" goods sector.

L_t^{kb} = Labor used in the fast-growing "capital" goods sector.

MC_t^{cbi} = Marginal cost in the slow-growing "consumption" goods sector.

MC_t^{kb} = Marginal cost in the fast-growing "capital" goods sector.

P_t^{cbi} = Price level in the slow-growing "consumption" goods sector.

P_t^{kb} = Price level in the fast-growing "capital" goods sector.

Q_t^{nr} = Price of installed non-residential capital.

Q_t^{r} = Price of installed residential capital.

Q_t^{cd} = Price of installed consumer durables capital.

R_t = Nominal interest rate.

$R_t^{nr,cbi}$ = The nominal rental rate on non-residential capital used in the slow-growing "consumption" goods sector.

$R_t^{nr,kb}$ = The nominal rental rate on non-residential capital used in the fast-growing "capital" goods sector.

R_t^{nr} = The aggregate nominal rental rate on non-residential capital.

R_t^{r} = The nominal rental rate on residential capital.

R_t^{cd} = The nominal rental rate on consumer durables capital.

U_t^{cbi} = The utilization rate of non-residential capital used in the slow-growing "consumption" goods sector.

U_t^{kb} = The utilization rate of non-residential capital used in the fast-growing "capital" goods sector.

W_t^{cbi} = The nominal wage in the slow-growing "consumption" goods sector.

W_t^{kb} = The nominal wage in the fast-growing "capital" goods sector.

X_t^{cbi} = Production in the slow-growing "consumption" goods sector.

X_t^{kb} = Production in the fast-growing "capital" goods sector.

X_t^{ch} = Production in the housing serives sector.

Z_t^{cbi} =

Z_t^{kb} =

Z_t^{m} =

$\Gamma_t^{x,cbi}$ =

$\Gamma_t^{x,kb}$ =

$\Gamma_t^{z,cbi}$ =

$\Gamma_t^{z,kb}$ =

$\Gamma_t^{z,m}$ =

$\Theta_t^{l,cbi}$ = The elasticity of subsitution between the differentiated labor inputs into the production of the economy's slow-growing "consumption" good.

$\Theta_t^{l,kb}$ = The elasticity of subsitution between the differentiated labor inputs into the production of the economy's fast-growing "capital" good.

$\Theta_t^{x,cbi}$ = The elasticity of subsitution between the differentiated intermediate inputs in the slow-growing "consumption" goods sector.

$\Theta_t^{x,kb}$ = The elasticity of subsitution between the differentiated intermediate inputs in the fast-growing "capital" goods sector.

Λ_t^{r} = The marginal utility of residential capital.

Λ_t^{cd} = The marginal utility of durable goods.

Λ_t^{cnn} = The marginal utility of non-durable goods and non-housing services consumption.

$\Lambda_t^{l,cbi}$ = The marginal dis-utility of supplying labor in the slow-growing "consumption" goods sector.

$\Lambda_t^{l,kb}$ = The marginal dis-utility of supplying labor in the fast-growing "capital" goods sector.

Ξ_t^{cnn} = Consumer non-durable goods and non-housing services consumption preference shock.

Ξ_t^{cd} = Consumer durable capital stock preference shock.

Ξ_t^{r} = Residential capital stock preference shock.

Ξ_t^{l} = Labor supply preference shock.

$\Pi_t^{p,gdp}$ = The inflation rate of the GDP deflator.

$\Pi_t^{p,pce}$ = The inflation rate of the PCE deflator

$\Pi_t^{p,cbi}$ = The inflation rate for prices in the slow-growing "consumption" goods sector.

$\Pi_t^{p,kb}$ = The inflation rate for prices in the fast-growing "capital" goods sector.

$\Pi_t^{p,cg}$ = The inflation rate for the price of government output (exogenous)

$\Pi_t^{w,cbi}$ = The inflation rate of nominal wages in the slow-growing "consumption" goods sector.

$\Pi_t^{w,kb}$ = The inflation rate of nominal wages in the fast-growing "capital" goods sector.

E Definitions of Stationary Model Variables

$\widetilde{E}_t^{nr} = \frac{E_t^{nr}}{Z_t^m Z_t^{kb}}$

$\widetilde{E}_t^{r} = \frac{E_t^{r}}{Z_t^m (Z_t^{kb})^{\alpha} (Z_t^{cbi})^{1-\alpha}}$

$\widetilde{E}_t^{cd} = \frac{E_t^{cd}}{Z_t^m Z_t^{kb}}$

$\widetilde{E}_t^{cnn} = \frac{E_t^{cc}}{Z_t^m (Z_t^{kb})^{\alpha} (Z_t^{cbi})^{1-\alpha}}$

$\widetilde{E}_t^{ch} = \frac{E_t^{ch}}{Z_t^m (Z_t^{kb})^{\alpha} (Z_t^{cbi})^{1-\alpha}}$

$\widetilde{K}_t^{u,nr,cbi} = \frac{K_t^{u,nr,cbi}}{Z_t^m Z_t^{kb}}$

$\widetilde{K}_t^{u,nr,kb} = \frac{K_t^{u,nr,kb}}{Z_t^m Z_t^{kb}}$

$\widetilde{K}_t^{nr,cbi} = \frac{K_t^{nr,cbi}}{Z_t^m Z_t^{kb}}$

$\widetilde{K}_t^{nr,kb} = \frac{K_t^{nr,kb}}{Z_t^m Z_t^{kb}}$

$\widetilde{K}_{t+1}^{nr} = \frac{K_t^{nr}}{Z_t^m Z_t^{kb}}$

$\widetilde{K}_{t+1}^{r} = \frac{K_t^{r}}{Z_t^m (Z_t^{kb})^{\alpha} (Z_t^{cbi})^{1-\alpha}}$

$$\widetilde{K}_{t+1}^{cd} = \frac{K_t^{cd}}{Z_t^m Z_t^{kb}}$$

$$\widetilde{MC}_t^{cbi} = \frac{MC_t^{cbi}}{P_t^{cbi}}$$

$$\widetilde{MC}_t^{kb} = \frac{MC_t^{kb}}{P_t^{cbi}} \left(\frac{Z_t^{kb}}{Z_t^{cbi}}\right)^{1-\alpha}$$

$$\widetilde{P}_t^{kb} = \frac{P_t^{kb}}{P_t^{cbi}} \left(\frac{Z_t^{kb}}{Z_t^{cbi}}\right)^{1-\alpha}$$

$$\widetilde{Q}_t^{nr} = \frac{Q_t^{nr}}{P_t^{cbi}} \left(\frac{Z_t^{kb}}{Z_t^{cbi}}\right)^{1-\alpha}$$

$$\widetilde{Q}_t^{r} = \frac{Q_t^{r}}{P_t^{cbi}}$$

$$\widetilde{Q}_t^{cd} = \frac{Q_t^{cd}}{P_t^{cbi}} \left(\frac{Z_t^{kb}}{Z_t^{cbi}}\right)^{1-\alpha}$$

$$\widetilde{R}_t^{nr,cbi} = \frac{R_t^{nr,cbi}}{P_t^{cbi}} \left(\frac{Z_t^{kb}}{Z_t^{cbi}}\right)^{1-\alpha}$$

$$\widetilde{R}_t^{nr,kb} = \frac{R_t^{nr,kb}}{P_t^{cbi}} \left(\frac{Z_t^{kb}}{Z_t^{cbi}}\right)^{1-\alpha}$$

$$\widetilde{R}_t^{nr} = \frac{R_t^{nr}}{P_t^{cbi}} \left(\frac{Z_t^{kb}}{Z_t^{cbi}}\right)^{1-\alpha}$$

$$\widetilde{R}_t^{r} = \frac{R_t^{r}}{P_t^{cbi}}$$

$$\widetilde{R}_t^{cd} = \frac{R_t^{cd}}{P_t^{cbi}} \left(\frac{Z_t^{kb}}{Z_t^{cbi}}\right)^{1-\alpha}$$

$$\widetilde{W}_t^{cbi} = \frac{W_t^{cbi}}{P_t^{cbi}} \cdot \frac{1}{Z_t^m (Z_t^{kb})^{\alpha} (Z_t^{cbi})^{1-\alpha}}$$

$$\widetilde{W}_t^{kb} = \frac{W_t^{kb}}{P_t^{cbi}} \cdot \frac{1}{Z_t^m (Z_t^{kb})^{\alpha} (Z_t^{cbi})^{1-\alpha}}$$

$$\widetilde{X}_t^{cbi} = \frac{X_t^{cbi}}{Z_t^m (Z_t^{kb})^{\alpha} (Z_t^{cbi})^{1-\alpha}}$$

$$\widetilde{X}_t^{kb} = \frac{X_t^{kb}}{Z_t^m Z_t^{kb}}$$

$$\widetilde{X}_t^{ch} = \frac{X_t^{ch}}{Z_t^m (Z_t^{kb})^{\alpha} (Z_t^{cbi})^{1-\alpha}}$$

$$\widetilde{\Lambda}_t^{r} == \Lambda_t^{r} Z_t^m (Z_t^{kb})^{\alpha} (Z_t^{cbi})^{1-\alpha}$$

$$\widetilde{\Lambda}_t^{cd} == \Lambda_t^{cd} Z_t^m Z_t^{kb}$$

$$\widetilde{\Lambda}_t^{cnn} = \Lambda_t^{cnn} Z_t^m (Z_t^{kb})^{\alpha} (Z_t^{cbi})^{1-\alpha}$$

Table 1: Calibrated Parameters

ψ	$\Theta^{cbi,p}$	$\Theta^{kb,p}$	$\Theta^{l,w}$	η^{nr}	η^{cd}	η^{r}	β	δ	δ_{cd}	δ_{ch}	$h^{*,gf}$	α
5	7	7	7	1	1	1	0.99	0.03	0.055	0.004	0.25	0.26

Table 2: Prior and Posterior Distributions

	Prior Distribution			Posterior Distribution				
Parameter	Type	Mean	S.D.	Mode	S.D.	10th perc.	50th perc.	90th perc.
h	B	0.500	0.122	0.794	0.055	0.717	0.794	0.862
h^{cd}	B	0.500	0.122	0.811	0.147	0.388	0.568	0.777
h^{r}	B	0.500	0.122	0.502	0.120	0.348	0.510	0.663
ν	G	2.000	1.000	2.443	0.915	2.124	3.168	4.537
r_{π}	N	2.000	1.000	3.686	0.488	3.021	3.570	4.277
r_{y}	N	0.500	0.400	0.208	0.029	0.170	0.209	0.243
$r_{\triangle\pi}$	N	0.500	0.400	-0.059	0.081	-0.163	-0.061	0.043
$r_{\triangle y}$	N	0.500	0.400	-0.091	0.028	-0.128	-0.095	-0.058
χ^{p}	G	2.000	1.000	2.217	0.673	2.191	2.951	3.878
χ^{H}	G	2.000	1.000	0.790	1.204	0.615	1.777	3.760
χ^{w}	G	2.000	1.000	0.696	0.691	0.634	1.271	2.384
χ^{nr}	G	2.000	1.000	0.660	0.242	0.548	0.817	1.160
χ^{cd}	G	2.000	1.000	0.191	0.159	0.296	0.521	0.708
χ^{r}	G	6.000	1.000	8.967	2.565	7.408	10.165	13.988
η^{H}	B	0.500	0.224	0.563	0.207	0.299	0.624	0.853
η^{p}	B	0.500	0.224	0.332	0.136	0.198	0.378	0.551
η^{w}	B	0.500	0.224	0.264	0.140	0.150	0.321	0.507
ρ_{R}	B	0.750	0.112	0.901	0.017	0.877	0.900	0.919
$\rho_{\xi,cnn}$	B	0.750	0.112	0.766	0.094	0.651	0.792	0.882
$\rho^{a,nr}$	B	0.750	0.112	0.887	0.031	0.853	0.893	0.929
$\rho^{a,cd}$	B	0.750	0.112	0.827	0.096	0.564	0.691	0.808
$\rho^{\xi,cd}$	B	0.750	0.112	0.793	0.112	0.590	0.753	0.882
$\rho^{\xi,r}$	B	0.750	0.112	0.790	0.109	0.591	0.755	0.875
$\rho^{gf,y}$	B	0.750	0.112	0.981	0.012	0.957	0.976	0.988
ρ^{L}	B	0.750	0.112	0.944	0.030	0.887	0.934	0.965
$\rho^{z,k}$	B	0.750	0.112	0.792	0.092	0.607	0.749	0.842
$\rho^{z,m}$	B	0.500	0.150	0.308	0.074	0.199	0.295	0.390
$\rho^{a,r}$	B	0.500	0.150	0.425	0.085	0.331	0.439	0.554
$\sigma_{\xi,b}$	I	3.000	2.000	1.755	0.404	1.441	1.849	2.470
$\sigma_{\xi,cd}$	I	3.000	2.000	2.457	2.658	2.095	3.704	7.699
$\sigma_{\xi,r}$	I	3.000	2.000	2.446	1.586	1.939	3.248	5.899
$\sigma_{gf,y}$	I	1.000	2.000	1.585	0.154	1.428	1.613	1.826
$\sigma_{\xi,l}$	I	3.000	2.000	2.357	1.067	2.303	3.342	5.033
σ_{R}	I	0.200	2.000	0.110	0.011	0.100	0.112	0.128
$\sigma_{z,k}$	I	0.500	2.000	0.410	0.202	0.356	0.539	0.848
$\sigma_{z,m}$	I	0.500	2.000	0.834	0.076	0.752	0.844	0.943
$\sigma_{\theta,y,cbi}$	I	0.500	2.000	0.522	0.155	0.492	0.668	0.899
$\sigma_{\theta,y,kb}$	I	0.500	2.000	0.416	0.157	0.317	0.486	0.716
$\sigma_{\theta,l}$	I	0.500	2.000	0.605	0.079	0.513	0.598	0.713
$\sigma_{a,r}$	I	4.000	2.000	8.241	2.581	6.509	9.082	13.237
$\sigma_{a,cd}$	I	2.000	2.000	1.855	1.746	2.487	4.740	7.050
$\sigma_{\xi,i}$	I	4.000	2.000	5.353	1.140	4.744	6.060	7.682

Table 3: Variance Decomposition

Shocks	Horizon	$\triangle y_t$	X_t^{cbi}	X_t^{kb}	L_t^{Agg}	Π_t^c	Π_t^{GDP}	R_t
$\epsilon_t^{\xi,cnn}$	1	0.01	0.15	0.02	0.03	0.00	0.00	0.01
	4	0.00	0.23	0.02	0.02	0.01	0.01	0.03
	12	0.00	0.27	0.02	0.01	0.02	0.01	0.03
$\epsilon_t^{\xi,cd}$	1	0.00	0.00	0.01	0.01	0.00	0.00	0.00
	4	0.00	0.00	0.01	0.01	0.00	0.00	0.01
	12	0.00	0.00	0.00	0.00	0.00	0.00	0.01
$\epsilon_t^{\xi,r}$	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	4	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	12	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$\epsilon_t^{h,gf}$	1	0.23	0.00	0.01	0.02	0.01	0.01	0.07
	4	0.06	0.00	0.01	0.02	0.03	0.03	0.04
	12	0.04	0.00	0.00	0.01	0.03	0.02	0.01
$\epsilon_t^{\xi,l}$	1	0.01	0.00	0.02	0.03	0.03	0.03	0.00
	4	0.05	0.01	0.03	0.10	0.07	0.06	0.00
	12	0.05	0.10	0.06	0.35	0.05	0.04	0.00
ϵ_t^{R}	1	0.02	0.00	0.03	0.05	0.02	0.03	0.57
	4	0.01	0.00	0.02	0.04	0.07	0.06	0.09
	12	0.00	0.01	0.00	0.01	0.10	0.08	0.01
$\epsilon_t^{z,kb}$	1	0.00	0.00	0.18	0.10	0.01	0.06	0.03
	4	0.00	0.00	0.16	0.11	0.04	0.15	0.11
	12	0.13	0.05	0.08	0.05	0.06	0.25	0.09
$\epsilon_t^{z,m}$	1	0.46	0.72	0.02	0.23	0.26	0.32	0.02
	4	0.51	0.52	0.01	0.11	0.65	0.59	0.01
	12	0.56	0.35	0.00	0.00	0.69	0.54	0.00
$\epsilon_t^{\theta,y,cbi}$	1	0.00	0.00	0.00	0.01	0.64	0.48	0.11
	4	0.00	0.01	0.00	0.01	0.06	0.03	0.06
	12	0.00	0.01	0.00	0.00	0.00	0.00	0.01
$\epsilon_t^{\theta,y,kb}$	1	0.00	0.00	0.01	0.00	0.00	0.02	0.00
	4	0.00	0.00	0.01	0.01	0.00	0.00	0.00
	12	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$\epsilon_t^{\theta,l}$	1	0.00	0.00	0.00	0.01	0.02	0.02	0.00
	4	0.01	0.00	0.00	0.02	0.02	0.02	0.00
	12	0.00	0.01	0.00	0.02	0.00	0.00	0.00
$\epsilon_t^{a,r}$	1	0.01	0.04	0.00	0.01	0.00	0.00	0.01
	4	0.00	0.04	0.00	0.01	0.00	0.00	0.01
	12	0.00	0.03	0.00	0.00	0.00	0.00	0.01
$\epsilon_t^{a,cd}$	1	0.04	0.01	0.11	0.09	0.01	0.01	0.04
	4	0.02	0.01	0.07	0.08	0.03	0.03	0.12
	12	0.00	0.03	0.01	0.02	0.03	0.03	0.06
$\epsilon_t^{a,nr}$	1	0.19	0.07	0.60	0.40	0.00	0.01	0.12
	4	0.33	0.16	0.67	0.47	0.01	0.02	0.52
	12	0.21	0.15	0.82	0.52	0.01	0.02	0.78

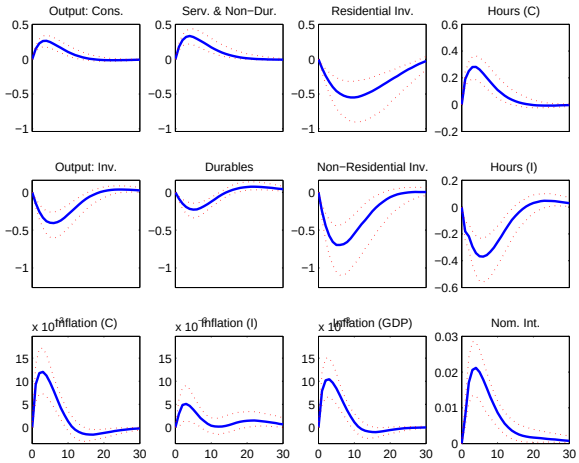


Figure 1: Impulse Responses: Preference Shock (Serv. & Non-Dur. Goods)

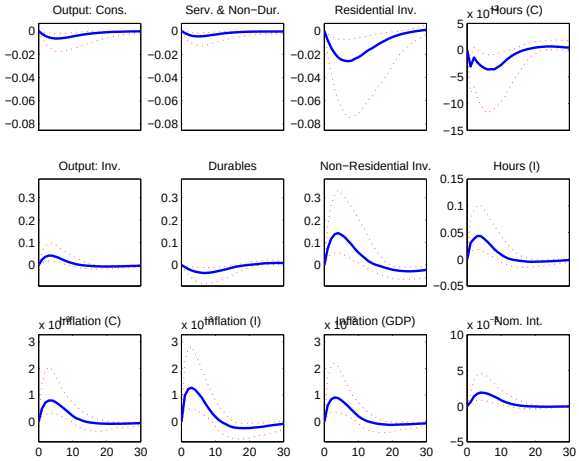


Figure 2: Impulse Responses: Preference Shock (Durable Goods)

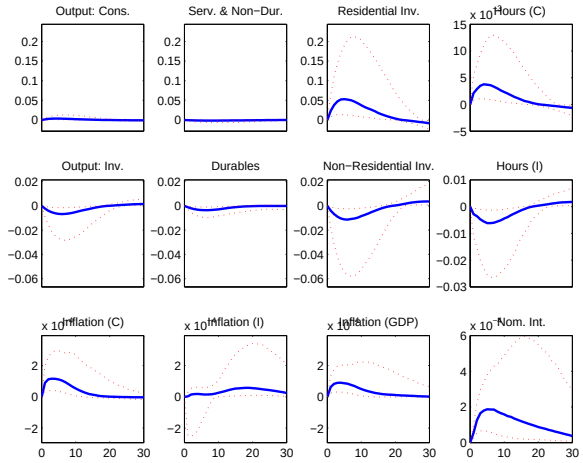


Figure 3: Impulse Responses: Preference Shock (Res. Goods)

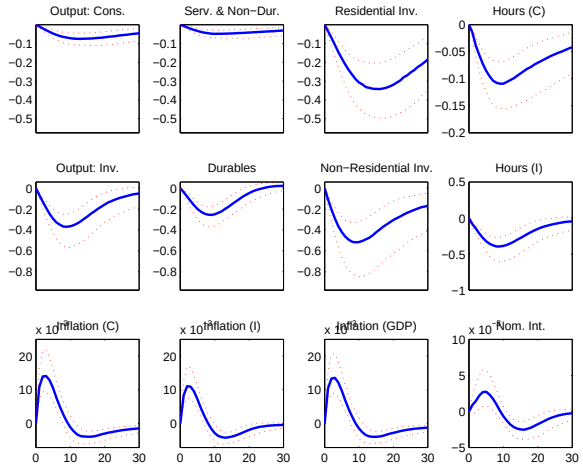


Figure 4: Impulse Responses: Labor Supply Shock

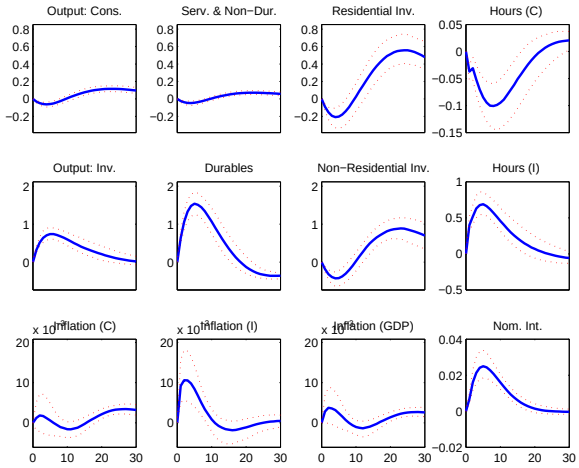


Figure 5: Impulse Responses: Efficiency Shock (Non-Res. Goods)

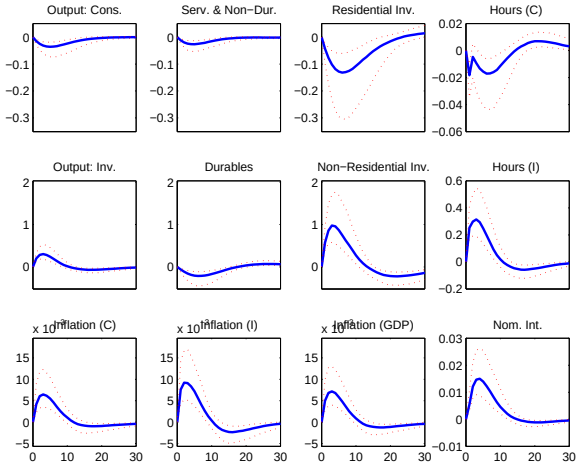


Figure 6: Impulse Responses: Efficiency Shock (Dur. Goods)

Figure 7: Impulse Responses: Efficiency Shock (Res. Goods)

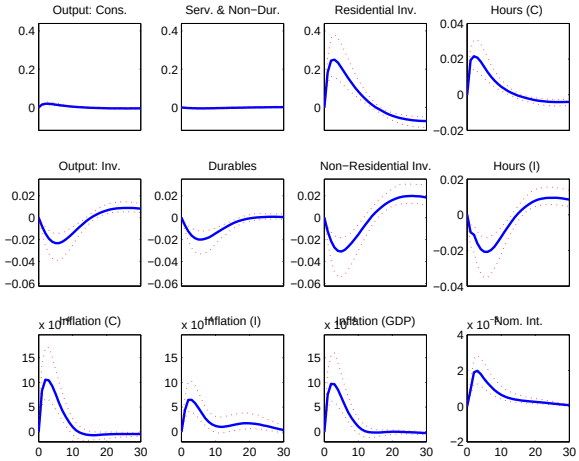


Figure 8: Impulse Responses: Autonomous Spending Shock

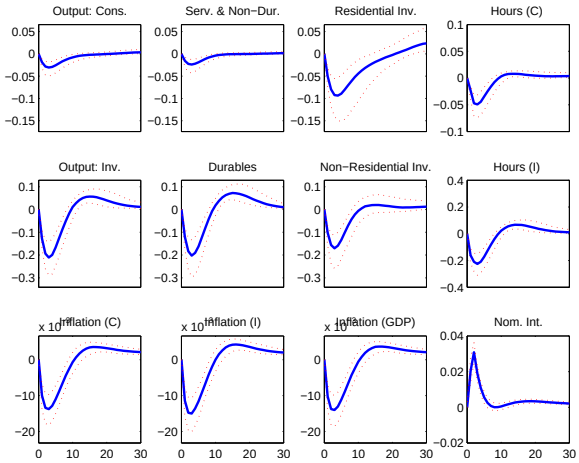


Figure 9: Impulse Responses: Price Markup Shock (CBI sector)

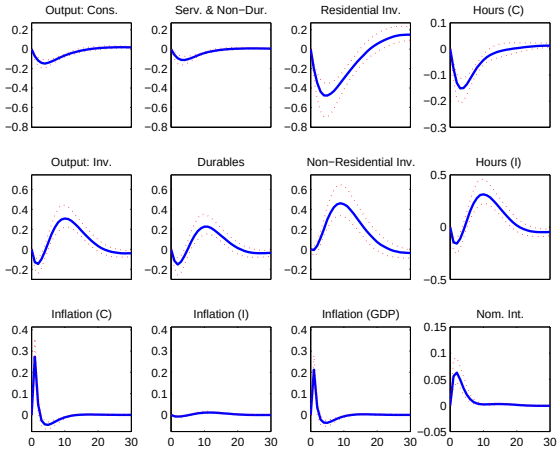


Figure 10: Impulse Responses: Price Markup Shock (BK sector)

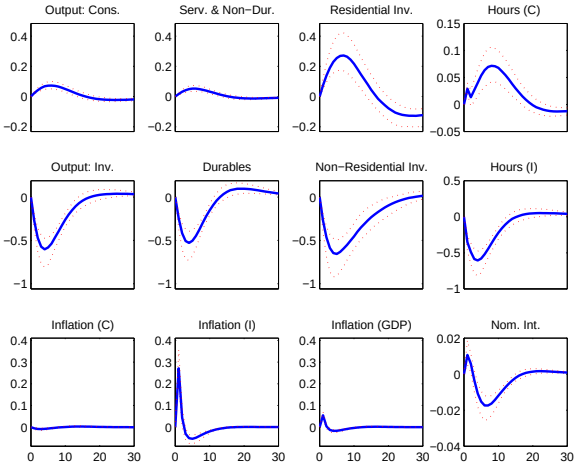


Figure 11: Impulse Responses: Permanent Technology Shock: TFP

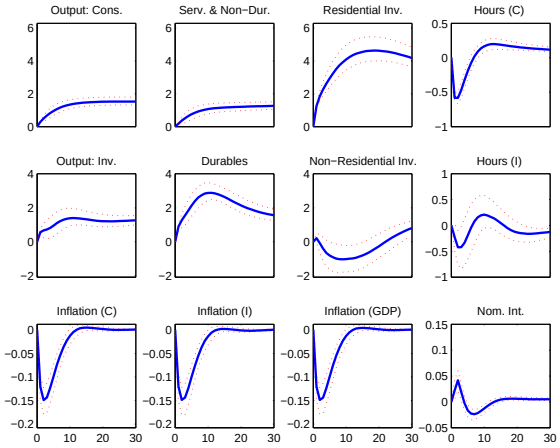
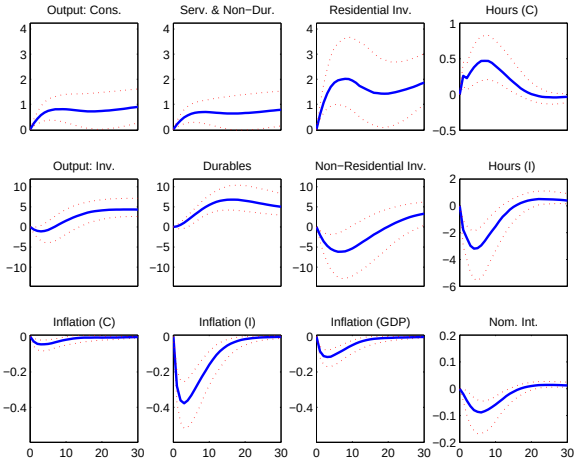


Figure 12: Impulse Responses: Permanent Technology Shock: Inv-Spec.



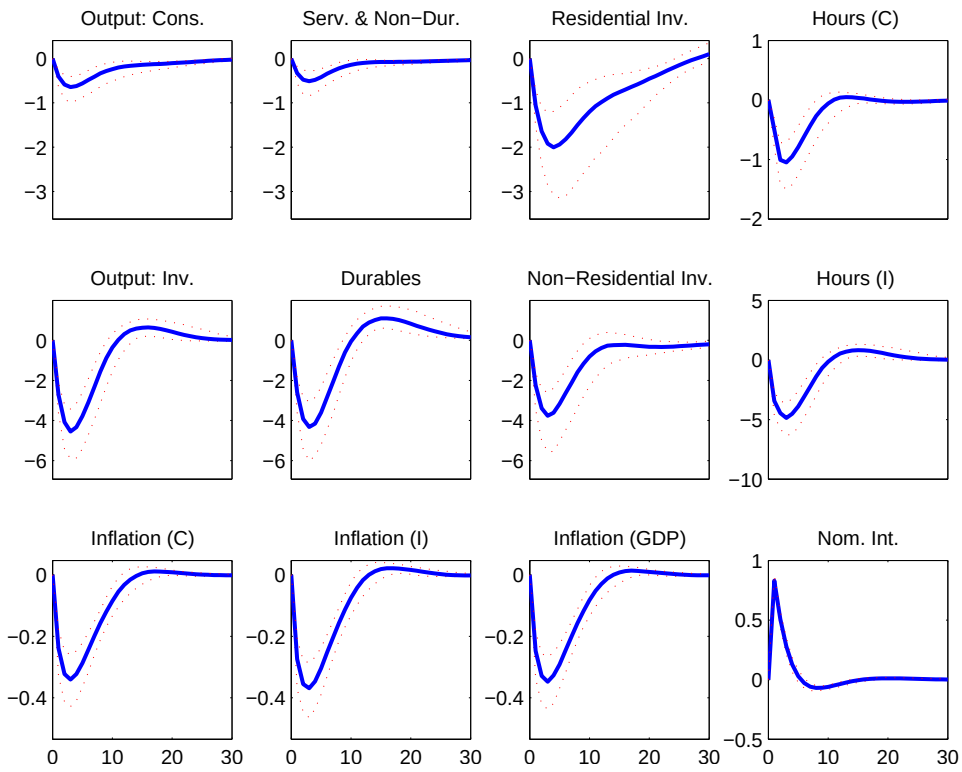
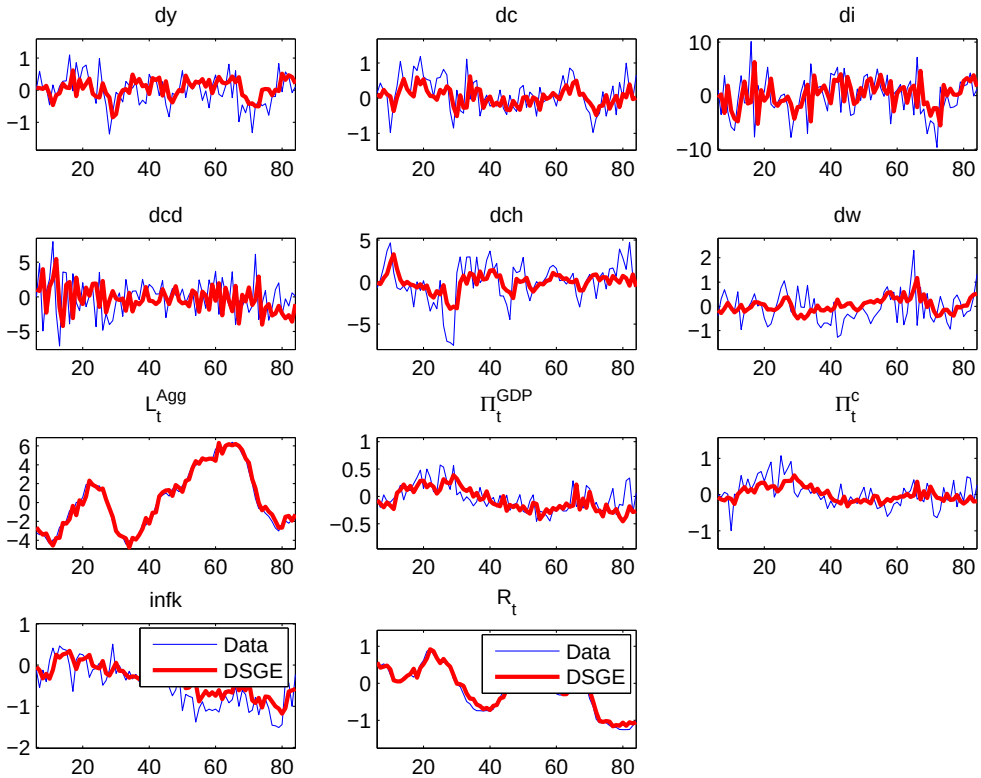


Figure 13: Impulse Responses: Monetary Policy Shock

Figure 14: Observable series: realized paths and one-step ahead forecasts.



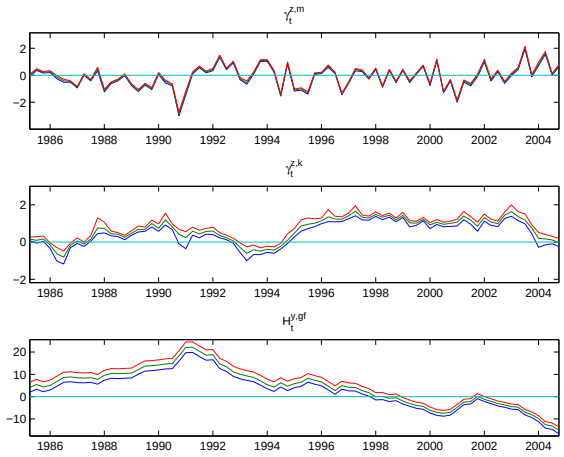


Figure 15: Smoothed Paths of Key Persistent Shocks

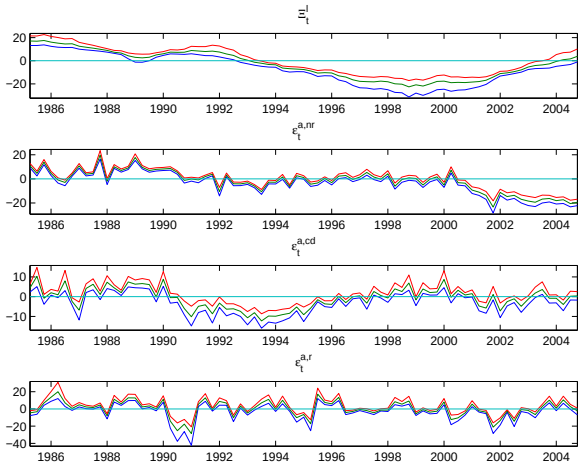


Figure 16: Smoothed Paths of Key Persistent Shocks II